Multivariate mixed-effects models for reflectance of forest trees

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Based on joint work with
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Outline

1. Background
2. Material
   - Empirical data
   - Structure of aerial image data
3. Methods
   - A simple mixed-effects model
   - Multivariate mixed-effects model
4. Results
5. Discussion

Background

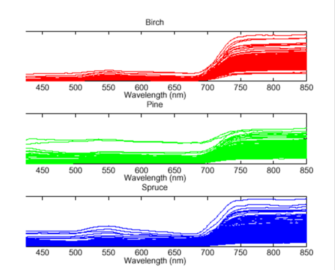
BACKGROUND –
The hard task

Pine
Spruce
Birch
Rowan
Oak?

- **Tree species is crucial** (allometry, canopy structure, wood properties, ecology,..)
- **Pulsed LiDAR** (*time-stamped photons*) - fair solutions by dense sampling, in single trees
- **Aerial passive imaging** (***free*** HS, RGN, RGBN, RGN-PAN ***photons***) for single trees and tree groups, spectral and also textural, human and machine vision since 1930s.

Background

Spectrally invariant features for species classification?



Jääskeläinen et al. 1992

The sad truth about trees

Separability – easier if classes are described by distinct, observable features. Trees show **high within-species** variation in spectral refl. (structure)

Observations – in forest we have shading, multiple scattering and occlusions, and the atmosphere constitutes an additional challenge.

Study material

- 20 partially overlapping strips collected by an aircraft-mounted line sensor. These strips are called (aerial) images
- The raw data was postprocessed to provide atmospherically corrected reflectance data on four channels: RED, GRN, BLU and NIR.
- $N = 15188$ dominant trees discernible in images formed the reference tree data (5914 Scots pines, 7105 Norway spruces, 2169 Birches)
- Airborne laser scanning data were used to map the individual trees on different images and find a certain tree on different images.
- The individual pixels within tree crowns were divided to sunlit and self-shaded pixels. The mean reflectances in these parts were analyzed separately
- For details, see Korpela et al. (2010; 2011).

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Structure of aerial image data on a forest

- Image data are hierarchical
  - Images within a geographical area
  - Recognized individual crowns within an image
  - Individual pixels within a tree crown
- The hierarchy may be an issue if it results in correlation among observations
  - Observations from a given image are similar due to e.g. the atmospheric correction effects
  - Trees of a given geographical area are similar to each other due to the structure of the landscape
  - Individual pixels in a tree are similar to each other due to tree-specific properties.

Such hierarchical datasets where the groups represent a sample from a population of groups are often modeled modeled using **mixed-effects models**

- Certain special cases are called variance component models, random effects models or random coefficient models.

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Nested and crossed groupings

Consider a data with grouping structure including two grouping factors, e.g. images, trees and pixels. The observation may be e.g. the (mean) reflectance of the object at a given wavelength (e.g. Near-Infrared, NIR)

- The grouping can be **nested** or **crossed**.
- In the **nested grouping**, the levels of grouping can ge hierarchically ordered
  - The members of a certain group at a lower level of hierarchy always belong to the same group at an upper hierarchical level.
  - *Pixels on crowns on non-overlapping aerial images*
- In the **crossed grouping**, the groups cannot be ordered hierarchically
  - members of a certain group may belong to several groups at other levels of grouping.
  - *Tree crowns on overlapping aerial images*
  - The same tree may be seen on several images
  - The same image includes several trees
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Simple mixed-effects model with crossed random effects

A simple mixed-effects model for two crossed groups and random constant can be written as

$$y_{it} = f(x_{it}|\boldsymbol{b}) + \alpha_i + \beta_t + \epsilon_{it} ,$$

where

- y_{it} is be the observed mean reflectance for the pixels of an individual crown t (of a given species) on image i
- $f(\boldsymbol{x}_{it}|\boldsymbol{b})$ is a function of fixed predictors (e.g. related to the view-illumination geometry)
- $\alpha_i \sim N(0, \sigma_\alpha^2)$ is a random image-effect
- $\beta_t \sim N(0, \sigma_\beta^2)$ is a random tree-effect
- $\epsilon_{it} \sim N(0, \sigma^2)$ is a random residual.

The model can also be written as

$$y_{it} = f(x_{it}) + e_{it} ,$$

by defining $e_{it} = \alpha_i + \beta_t + \epsilon_{it}$.

The random effects at different levels of grouping are independent, therefore $var(e_{it}) = \sigma_\alpha^2 + \sigma_\beta^2 + \sigma^2$
Also note: $cov(y_{it}, y_{i't'}) = 0$, $cov(y_{it}, y_{it'}) = \sigma_\beta^2$ and $cov(y_{it}, y_{i't}) = \sigma_\alpha^2$.

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Why random effects?

- The random effects divide the total variability into components arising from different levels of grouping.
- The division of the variance according to the level of grouping straightforwardly yields estimates for the covariance and correlation of the observations. These may be utilized in species classification (Mahalanobis distance).
- More reliable inference on the model parameters (i.e., the effect of potential fixed predictors on the ressponse)
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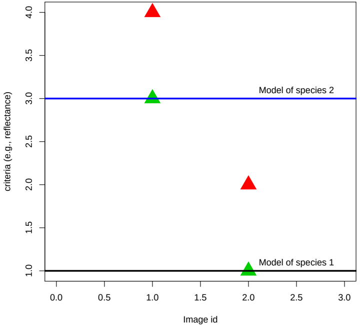
Image and tree effects in classification



- If the correlation arising from the image and tree effects would not be used, the tree shown by the red marks had dhorter distance to the model of species 2 whereas the other red tree had equally long distance to both models.
- But the order of the trees is similar on both images, therefore part of the difference in levels might be due to image effects.
- Or the red tree might be of species 1, just with a large tree effect.
- Estimated mixed-effects models provide variance-covariance structures that allow taking into account these effects in computing the Mahalanobis distance.

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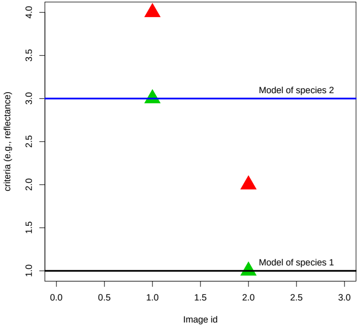

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Multivariate extension of the simple mixed-effects model

The reflectance may be observed for different wavelengths (RED, GRN, BLU, NIR) and separately for sunlit (SL) and self-shaded (SS) parts of the crown. Each of these 8 responses need a separate model, but the random effects and residuals of the model might be correlated. To allow this, a multivariate version of the model is specified as

$$\begin{aligned} y1_{it} &= f1(x_{it}|\boldsymbol{b}1) + \alpha 1_i + \beta 1_t + \epsilon 1_{it} \\ y2_{it} &= f2(x_{it}|\boldsymbol{b}2) + \alpha 2_i + \beta 2_t + \epsilon 2_{it} \\ &\vdots \\ y8_{it} &= f8(x_{it}|\boldsymbol{b}8) + \alpha 8_i + \beta 8_t + \epsilon 8_{it} \end{aligned} \quad (1)$$

where

- $(\alpha 1_i, \alpha 2_i, \ldots, \alpha 8_i)' = \boldsymbol{\alpha}_i \sim MVN(0, \boldsymbol{A}_{8\times 8})$ is a random vector of image-effects
- $(\beta 1_t, \beta 2_t, \ldots, \beta 8_t)' = \boldsymbol{\beta}_t \sim MVN(0, \boldsymbol{B}_{8\times 8})$ is a random vector of tree-effects
- $(\epsilon 1_{it}, \epsilon 2_{it}, \ldots, \epsilon 8_{it})' = \boldsymbol{\epsilon}_{it} \sim MVN(0, \boldsymbol{E}_{8\times 8})$ is a random vector of residuals.
- Model fitting (using e.g. Restricted Maximum Likelihood, REML) yields estimates $\widehat{\boldsymbol{b}}$, $\widehat{\boldsymbol{A}}$, $\widehat{\boldsymbol{B}}$ and $\widehat{\boldsymbol{E}}$
- Predictions of random effects $\widetilde{\alpha}_i$, and $\widetilde{\beta}_t$ is possible for all groups with observarions as well (also afterwards).

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Mahalanobis distance

- Let $\boldsymbol{y}_{it}$ be an observed vector (length=8) of the reflectances of one tree t on the 8 channels on one image i. The squared Mahalanobis distance between $\boldsymbol{y}_{it}$ and $\boldsymbol{\mu}_{it}$ is

$$d_{it}^2 = (\boldsymbol{y}_{it} - \boldsymbol{\mu}_{it})'(\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{E})^{-1}(\boldsymbol{y}_{it} - \boldsymbol{\mu}_{it})$$

  This distance takes into account the correlation of reflectance among different channels, and is (at least under multivariate normality of the reflectance data) in a way optimal for single tree on single image.

- A simple solution to account for the multiple images is to compute the mean of image-specific distances. This solution does not take into account the correlation due to tree effects and image effects.

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Mahalanobis distance for multiple images

- To compute the Mahalanobis distance between observations from multiple images, let $\boldsymbol{y}_{\cdot t} = (\boldsymbol{y}'_{1t}, \ldots, \boldsymbol{y}_{mt})$ be an observed vector (with length of $8m$) of the reflectances of one tree t on the 8 channels of m images. The squared Mahalanobis distance between $\boldsymbol{y}_{\cdot t}$ and $\boldsymbol{\mu}_{\cdot t}$ is

$$d_{\cdot t}^2 = (\boldsymbol{y}_{\cdot t} - \boldsymbol{\mu}_{\cdot t})' \boldsymbol{D}_{\cdot t}^{-1} (\boldsymbol{y}_{\cdot t} - \boldsymbol{\mu}_{\cdot t}) \,,$$

  where the $8m \times 8m$ variance-covariance matrix is

$$\boldsymbol{D}_{\cdot t} = \begin{bmatrix} \boldsymbol{A}+\boldsymbol{B}+\boldsymbol{E} & \boldsymbol{B} & \ldots & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{A}+\boldsymbol{B}+\boldsymbol{E} & & \boldsymbol{B} \\ \vdots & & \ddots & \vdots \\ \boldsymbol{B} & \boldsymbol{B} & \ldots & \boldsymbol{A}+\boldsymbol{B}+\boldsymbol{E} \end{bmatrix}$$

  This distance accounts for the correlation arising from the common tree effects

- Natural extensions for the distance that would take into account the correlation arising from the image effects (common for different trees of same image) would be possible as well.

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Mean reflectances

To-NADIR-adjusted reflectance factors (~dark-pixel method, ”HDRFs”) for sunlit and self-shaded crown patches

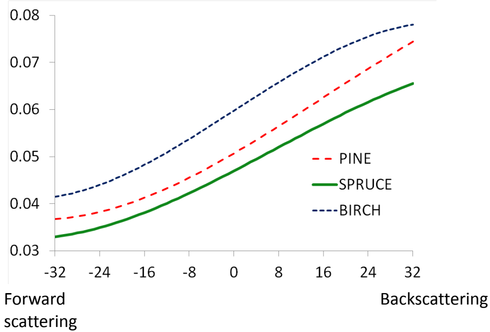
Illumination - Species	RED	GRN	BLU	NIR
SS-Pine	0.030	0.041	0.037	0.169
SS-Spruce	0.024	0.035	0.035	0.155
SS-Birch	0.031	0.044	0.038	0.227
SL-Pine	0.037	0.051	0.040	0.220
SL-Spruce	0.032	0.047	0.038	0.224
SL-Birch	0.043	0.060	0.042	0.322

Trees are dark, but show their bright side in NIR

Korpela, I., Heikkinen, V., Honkavaara, E., Rohrbach F., Tokola, T. 2011. Variation and anisotropy of reflectance of forest trees in radiometrically calibrated airborne line sensor images – implications for species classification in digital aerial images. Remote Sensing of Environment.

The role of the fixed part

I.e. Usable additional information, here?



Estimated variance components

Variance components, real data, 200 000 observations (%)

	sunlit	shade	sunlit	shade	sunlit	shade	sunlit	shade
Fixed (Xβ)-%	33	11	32	13	45	29	7	-0
Tree-%	42	42	43	41	18	13	62	64
Image-%	4	12	5	14	27	46	6	2
Residual-%	21	35	20	32	10	13	25	34
Total	100	100	100	100	100	100	100	100

* Fixed part: The anisotropy trends explained SL >> SS, BLU > GRN > RED > NIR. In NIR, anisotropy is low.

* Tree-effect: The correlations are strong, both for SL and SS. A bright tree is bright across views and bands. In NIR > 60% of variance explained!!

* Image-effect: Substantial in BLU, SS > SL. Includes effects from solar elevation changes (07-09 GMT), atmospheric correction errors.

Ilkka Korpela, Oct 2012

Classification accuracy

- The correlations among the multiple responses were strong, therefore using them in the distance is justified
- Using the simple mean over image-specific distances provided improved classification accuracy with increasing number of images
- Tentative results on the use of the Mahalanobis distance in the classification provided much stronger improvement compared to the simple mean method

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Discussion

- Mixed-effects models are useful tools for analyzing grouped datasets in different contexts.
- The benefit from the use of mixed-effects models depends on the application.
- In classification problems, the mixed-effects models provide a justified means to estimate the required varaince-covariance matrix (See e.g. Fieuws et al 2008).
- If the aim is to test treatment effects, taking into account the correlation structure leads to more reliable inference (p-values)
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- In classification problems, the mixed-effects models provide a justified means to estimate the required varaince-covariance matrix (See e.g. Fieuws et al 2008).
- If the aim is to test treatment effects, taking into account the correlation structure leads to more reliable inference (p-values)
- If the aim is prediction, then possibility to make predictions for different levels of grouping may be crucial, given that observartion(s) from teh group in question is available.

Discussion

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Thank you for your interest!

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