Height-Diameter curves from longitudinal data

Lauri Mehtätalo
lauri.mehtatalo@yale.edu

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Outline of the presentation

The aim of this study

The aim was to estimate such a Height-Diameter model that could be used for prediction using the fixed predictors and height sample tree measurements from various points in time.
More specifically, the model should be able to

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3. use old height sample trees to improve the prediction of current point in time,
4. predict the H-D curve in the future,

Data

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- We are using Norway spruces, which is a shade tolerant tree species
- Data inclusdes 249 stands, 1-4 measurement occasions with 5 years intervals
- A total of 18056 height measurements, 3–49 trees per stand

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I reparameterized D and linearized the model to get

$$\ln(H_{kti}) = A_{kt} + B_{kt}x_{kti} + \epsilon_{kti} \quad (1)$$

The reparameterization of D_{kti} was

$$x_{kti} = \frac{(D_{kti} + \lambda)^{-C} - (DGM + 10 + \lambda)^{-C}}{(30 + \lambda)^{-C} - (10 + \lambda)^{-C}},$$

where $\lambda = 7cm$ and $C = 1.564$.

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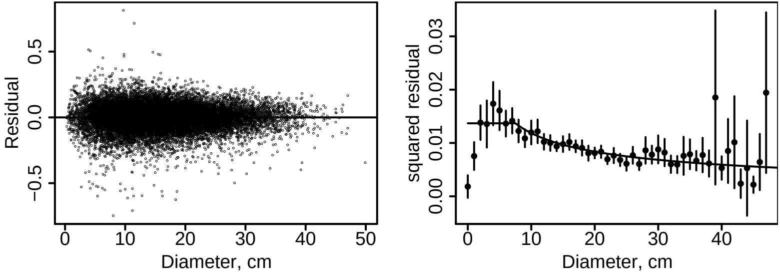
Interpretations

A is the expected logarithmic height of thickest trees in the stand
B is the difference in $\ln(H)$ between trees of diameters 30 and 10 cm

The variance function

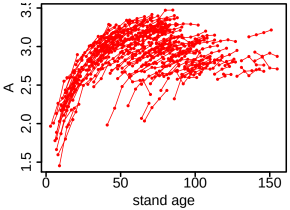
In a single stand, the variance of height decreases with increasing diameter

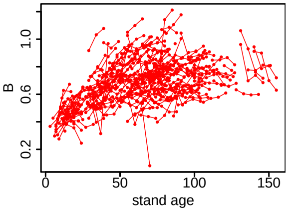


The variance function I used was

$$\mathrm{var}(\epsilon_{kti}) = \sigma^2 \left(\max(D_{kti}, 7.5) \right)^{-2\delta} \tag{2}$$

Separate fits





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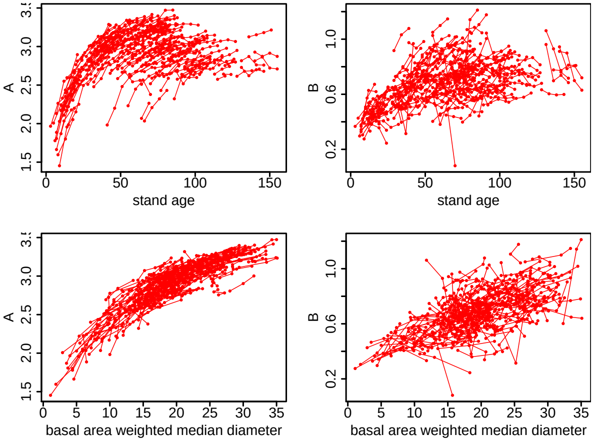


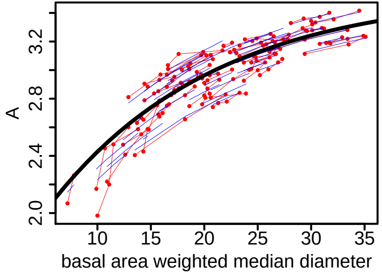
Trend functions

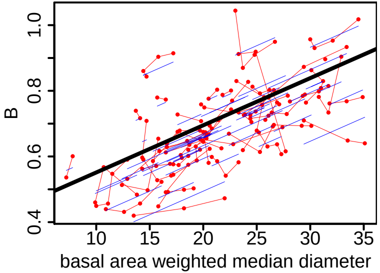
A: Champman-Richards

$$A_{kt} = p_{1a} + p_{2a}\left(1 - \exp(-p_{3a}DGM_{kt})\right)^{p_{4a}} + \alpha_k + \alpha_{kt} \quad (3)$$

B: linear

$$B_{kt} = p_{1b} + p_{2b}DGM_{kt} + \beta_k + \beta_{kt}$$





Complete model

I wrote the trend functions to model 1 to obtain

$$\begin{aligned} \ln(H_{kti}) &= p_{1a} + \alpha_k + \alpha_{kt} + p_{2a} z_{kt} \\ &\quad + (p_{1b} + \beta_k + \beta_{kt} + p_{2b} DGM_{kt}) x_{kti} + \epsilon_{kti} \,, \end{aligned}$$

where z_{kt} is the nonlinear part of trend function 3, which is treated as a transformation of DGM_{kt}:

$$z_{kt} = \left(1 - \exp(-0.0651 DGM_{kt})\right)^{0.999}$$

α_k and β_k are correlated, normal, stand specific random parameters, α_{kt} and β_{kt} are correlated, normal, measurement occasion specific random parameters, and ϵ_{kti} are normal tree level errors with variance as given in 2.

Estimates

The parameter estimates and their standard errors were as follows

	p_{1a}	p_{2a}	p_{1b}	p_{2b}
estimate	1.40	2.15	0.384	0.0157
standard error	0.026	0.035	0.017	0.001

$$
\begin{aligned}
\mathrm{var}\begin{bmatrix} \alpha_k \\ \beta_k \end{bmatrix} &= \begin{bmatrix} 0.108^2 & (0.269) \\ 0.0028 & 0.0958^2 \end{bmatrix} \\
\mathrm{var}\begin{bmatrix} \alpha_{kt} \\ \beta_{kt} \end{bmatrix} &= \begin{bmatrix} 0.0168^2 & (-0.681) \\ -0.0003 & 0.0223^2 \end{bmatrix} \\
\mathrm{var}(\epsilon_{kti}) &= 0.401^2 \left(\max(D_{kti}, 7.5) \right)^{-1.068}
\end{aligned}
$$

Prediction of random effects

The sample tree heights of a new stand can be described by

$$\mathbf{y} = \boldsymbol{\mu} + \mathbf{Zb} + \boldsymbol{\epsilon}\,,$$

where
$\mathbf{y}$ includes the observed sample tree heights,
$\boldsymbol{\mu}$ is the fixed part,
$\mathbf{b} = \begin{pmatrix} \alpha_k & \beta_k & \alpha_{k1} & \beta_{k1} & \alpha_{k2} & \beta_{k2} & \ldots \end{pmatrix}'$ includes the random effects,
$\mathbf{Z}$ is the corresponding design matrix, and
$\boldsymbol{\epsilon}$ includes the residuals.
Let us denote $\mathrm{var}(\mathbf{b}) = \mathbf{D}$ and $\mathrm{var}(\boldsymbol{\epsilon}) = \mathbf{R}$.

Utilizing the model

The variances and covariances between random effects and observed heights can be written as

$$\begin{bmatrix} \mathbf{b} \\ \mathbf{y} \end{bmatrix} \sim \left(\begin{bmatrix} \mathbf{0} \\ \boldsymbol{\mu} \end{bmatrix}, \begin{bmatrix} \mathbf{D} & \mathbf{DZ}' \\ \mathbf{ZD} & \mathbf{ZDZ}' + \mathbf{R} \end{bmatrix} \right)$$

The Empirical Best Linear Unbiased predictor of random effects is

$$\widehat{\mathbf{b}} = \mathbf{DZ}'(\mathbf{ZDZ}' + \mathbf{R})^{-1}(\mathbf{y} - \boldsymbol{\mu}) \,.$$

and the variance of prediction errors is

$$\mathrm{var}(\widehat{\mathbf{b}} - \mathbf{b}) = \mathbf{D} - \mathbf{DZ}'(\mathbf{ZDZ}' + \mathbf{R})^{-1}\mathbf{ZD}$$

Example

Heights of one tree was measured 5 years ago and 2 trees at this year. The matrices and vectors are

$$\boldsymbol{\mu} = \begin{bmatrix} 2.59 \\ 2.11 \\ 2.99 \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} 2.77 \\ 2.35 \\ 3.19 \end{bmatrix}$$

$$\mathbf{Z} = \begin{bmatrix} 1 & -0.36 & 1 & -0.36 & 0 & 0 \\ 1 & -1.22 & 0 & 0 & 1 & -1.22 \\ 1 & 0.058 & 0 & 0 & 1 & 0.058 \end{bmatrix} \quad \mathbf{R} = \begin{bmatrix} 0.008 & 0 & 0 \\ 0 & 0.016 & 0 \\ 0 & 0 & 0.004 \end{bmatrix}$$

$$\mathbf{b} = \begin{bmatrix} \alpha_k \\ \beta_k \\ \alpha_{k1} \\ \beta_{k1} \\ \alpha_{k2} \\ \beta_{k2} \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0.0118 & 0.0028 & 0 & 0 & 0 & 0 \\ 0.0028 & 0.0092 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0003 & 0.0004 & 0 & 0 \\ 0 & 0 & 0.0004 & 0.0005 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0003 & 0.0004 \\ 0 & 0 & 0 & 0 & 0.0004 & 0.0005 \end{bmatrix}$$

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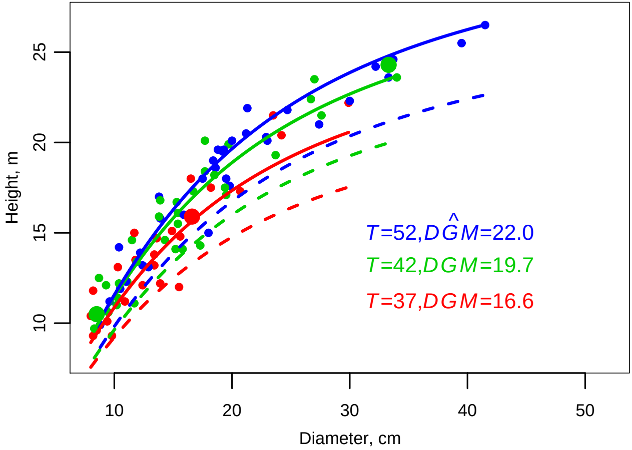
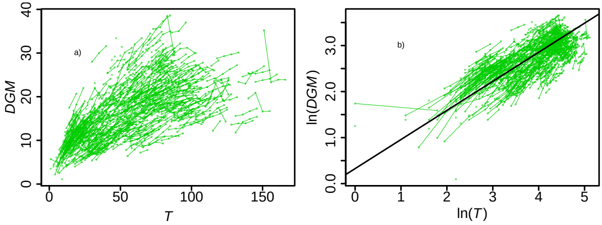

T=52,DĜM=22.0
T=42,DGM=19.7
T=37,DGM=16.6
Height, m
Diameter, cm
10
15
20
25
10
20
30
40
50

Model for DGM



The model for *DGM* was

$$\ln(DGM_{kt}) = u + v \ln T_{kt} + u_k + e_{kt} \; ,$$

where $u = 0.33$, $v = 0.63$, $\mathrm{var}(u_k) = 0.32^2$ and $\mathrm{var}(e_{kt}) = 0.069^2$.

Final remarks

- The mixed modeling for longitudinal data was a good approach in this context. It makes it possible to flexibly move forward and backward in time, and to utilize mesurements from different points in time in prediction.

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- The mixed modeling for longitudinal data was a good approach in this context. It makes it possible to flexibly move forward and backward in time, and to utilize mesurements from different points in time in prediction.
- It is very important to plot predictions and residuals in different ways to find good model formulae. Here it helped me to find how parameter A developes over time.
- The development of *H-D* curves was best modeled as a function of a variable describing the mean (or median) size of trees in the stand, instead of stand age. That could be a good approach also in other models.
- R-sowtware has many good properties that make this kind of analysis easy in R: function 'unique', packages 'nlme' and 'lme', possibility to handle several datasets and models at same time.