Finding hidden trees in remote sensing of forests by using stochastic geometry, sequential spatial point processes and the HT-like estimator

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Outline







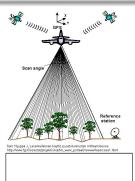


Conclusions

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Laser scanning in forest inventories



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Airborne laser scanning

- A laser scanner measures the distance from an aircraft to forest canopy
- Some pulses return from canopies, others from forest floor
- Produces point-wise measurements of canopy height, which allow detection of individual tree crowns.

Terrestrial laser scanning

- A laser scanner, installed on a tripod, rotates 360⁰ and measures the distances to the closest obstacles with a very high pulse density.
- Individual tree stems can be detected from the point cloud.

In both cases

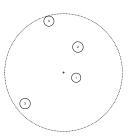
- The detected trees can be ordered according to their shortest distance to the censor
- Trees can be hidden if they are located behind "earlier" trees.



Assume that TLS is used to detect the stem discs at the height of 1.3 meters above the ground. The *n* observed trees out of the all *N* trees within a circular sample plot are ordered according to the distance r_i to the plot center, which is for convenience at the origin. Consider the total of variable m, $\tau = \sum_{i=1}^{N} m_i$. The Horvitz-Thompson like estimator is

$$\widehat{\tau}_m = \sum_{i=1}^n \frac{m_i}{p_i},$$

where the inclusion probability p_i is called detectability.



For tree 1, p₁ = 1.

Assuming that a tree is detected if the center point is visible to the scanner and CSR of tree locations, the detectabilities for the latter trees are computed using

$$p_i = 1 - \frac{\left| \left(\bigcup_{j=1}^{i-1} A_j \right) \cap S(r_i) \right|}{|S(r_i)|},$$

where A_j is the gray shaded area behind tree *j* and $S(r_j)$ is a origin-centered circle of radius r_j .^{*a*}

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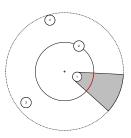
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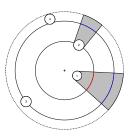
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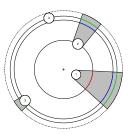
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- For tree 1, $p_1 = 1$.
- Assuming that a tree is detected if the center point is visible to the scanner and CSR of tree locations, the detectabilities for the latter trees are computed using

$$p_i = 1 - \frac{\left| \left(\bigcup_{j=1}^{i-1} A_j \right) \cap S(r_i) \right|}{|S(r_i)|},$$

where A_i is the gray shaded area behind tree *j* and $S(r_i)$ is a origin-centered circle of radius r_i.^a

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Expected value of the estimator under CSR

Denote by S_j that part of the r_j -radius circle that is hidden behind earlier trees, by S_j^c the part that is not hidden, and their relative lengths by $|S_j|$ and $|S_j^c|$ so that $|S_j| + |S_j^c| = 1$. Write the HT-like estimator as

$$\widehat{\tau}_m = \sum_{i=1}^N \frac{m_i I_i}{p_i}$$

where I_i ia an indicator that specifies whether tree *i* was detected. Now

$$\mathrm{E}(\widehat{\tau}_m) = \sum_{i=1}^N \frac{m_i \mathrm{E}(I_i)}{p_i}.$$

If tree locations are generated by the homogeneous Poisson process, then $E(I_i) = 1 \times |S_i^c| + 0 \times |S_i| = p_i$ and

$$\mathrm{E}(\widehat{\tau}_m) = \sum_{i=1}^N m_i = \tau_m$$

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An unbiased estimator of the classical Horvitz-Thompson estimator is

$$\widehat{\operatorname{var}}(\widehat{\tau}_m) = \sum_{i=1}^n \left(\frac{1}{p_i^2} + \frac{1}{p_i}\right) m_i^2 + 2\sum_{i=1}^n \sum_{j>i} \left(\frac{1}{p_i p_j} - \frac{1}{p_{ij}}\right) m_i m_j,$$

where p_{ij} is the joint inclusion probability of trees *i* and *j*. In our sequential construction

$$p_{ij} = p_{j|i}p_i = p_ip_j$$

and therefore the variance can be estimated by

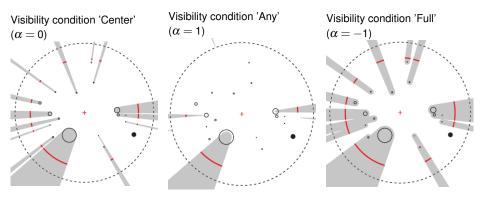
$$\widehat{\operatorname{var}}\left(\widehat{\tau}_{m}\right)=\sum_{i=1}^{n}\left(\frac{1}{p_{i}^{2}}+\frac{1}{p_{i}}\right)m_{i}^{2}.$$

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Generalizations to other detection conditions

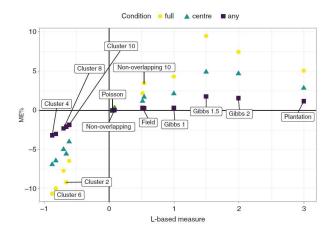
The previous formulation assumed that the center point is visible to the scanner. Extensions are based on applying a erosion/dilation using a buffer of width $|\alpha|r_i$ to the shadowed area before intersecting with the r_i -radius circle:





Bias vs spatial pattern

Evaluation was based on process-simulated plots and mapped large sample plots.



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Coverage of approximate confidence intervals

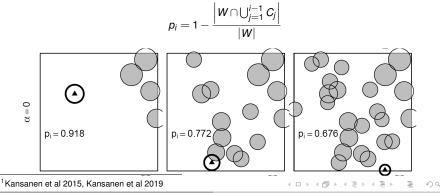
		Interval		
Data	Condition	90%	95 %	99%
Field	Full	94.0	97.5	99.5
	Center	94.1	97.4	99.3
	Any	93.4	96.5	98.8
Poisson	Full	90.0	94.9	98.7
	Center	89.9	94.5	98.7
	Any	90.5	94.9	98.4

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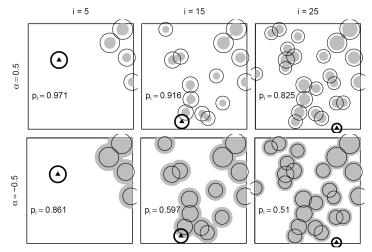


Application to the aerial case

- ALS is used to detect the tree crowns (= their projections to the ground C_i).
- The *n* observed trees within a sample plot window *W* are ordered according to the distance to the censor (= height from largest to smallest).
- We use the Horvitz-Thompson like estimator $\widehat{\tau_m} = \sum_{i=1}^n \frac{m_i}{p_i}$ to estimate $\tau = \sum_{i=1}^N m_i$.
- Assuming that a tree is detected if the center point is visible to the scanner and tree locations follow CSR, the detectability for tree *i* is ¹



Extension to other detection conditions



Dilate ($\alpha < 0$) or erode ($\alpha > 0$) using $|\alpha|r_i$ wide buffer, where $0 \le |\alpha| \le 1$.



The effect of spatial pattern of tree locations

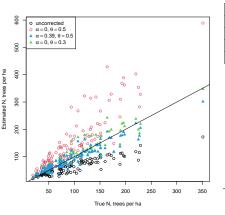
- The parameter α could be derived based on the applied tree detection algorithm.
- It can also be estimated empirically e.g. by matching a field-measured number of trees on the HT-like estimate in a training data set, and using the resulting estimate as a fixed known parameter thereafter. In that case, α simultaneously models the departure from CSR and detection condition.
- A sequential spatial point process (SSPP) model could be used to model the spatial pattern explicitly ². The model will be presented later today by Adil Yazigi.
- The SSPP model is used to predict the density in the hidden parts relative to that in the visible part as $\frac{\theta}{1-\theta}$. The value $\theta = 0.5$ corresponds to CSR.
- Thereafter the population total can be estimated using

$$\hat{\tau}_m = \sum_{i=1}^n m_i + \frac{\theta}{1-\theta} \sum_{i=1}^n \left[\left(\frac{1}{\rho_i} - 1 \right) m_i \right].$$

- In our empirical training data,
 - the mean of estimated α when $\theta = 0.5$ was $\hat{\alpha} = 0.39$
 - the mean of estimated θ when $\alpha = 0$ was $\hat{\theta} = 0.30$

²Yazigi et al. Revised MS

Tentative results with empirical data



Method		bias, %	RMSE, %
uncorrected		-35	48.8
$\alpha = 0,$	$\theta = 0.5$	57	80.9
$\alpha = 0.39,$	$\theta = 0.5$	-3.5	20.0
$\alpha = 0,$	$\theta = 0.30$	3.5	22.5

- 111 square 30 by 30 meter plots from Eastern Finland
- Field-measured tree locations, diameters and heights were available
- ALS data were used to detect individual trees using an ITD algorithm ^a

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^aLähivaara et al 2014

Conclusions

- A sequential construction of trees is highly useful in ALS inventories, where trees hide behind each others.
- Wethods are connected with distance sampling. Difference to classical HT-estimator arises from the model-based estimation of the inclusion probability (detectability).
- (a) In ALS, the order is based on tree size, whereas in TLS the distance and size are unrelated (but small trees still may hide more easily than the large trees if $\alpha \neq 0$).
- In the ALS-application, a regression approach where the parameters α and/or θ are predicted using ALS point cloud features ³ might result in further improvements of the method.
- In the TLS application, improvements might be obtained by estimating the point process based on the observed trees and conditioning estimation on it.
- Implementations of the proposed algorithms are available in R-package lmfor.

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Bibliography

- Häbel, Henrike; Balazs, Andras; Myllymäki, Mari. 2021. Spatial analysis of airborne laser scanning point clouds for predicting forest structure. Mathematical and Computational Forestry Natural Resource Sciences 13(1): 15-28.
- Kansanen, K., Vauhkonen, J., Lähivaara, T., Mehtätalo, L. 2016. Stand density estimators based on individual tree detection and stochastic geometry. Canadian Journal of Forest Research 46(11): 1359-1366. 10.1139/cjfr-2016-0181.
- Kansanen, K., J. Vauhkonen, T. Lähivaara, A. Seppänen, M. Maltamo, and L. Mehtätalo. 2019. Estimating forest stand density and structure using Bayesian individual tree detection, stochastic geometry, and distribution matching. ISPRS Journal of Photogrammetry and Remote Sensing 152:66-78.
- Kansanen, K., Packalen, P., Maltamo, M. and Mehtätalo, L. 2020. Horvitz-Thompson-like estimation with distance-based detection probabilities for circular plot sampling of forests. Biometrics (early view). https://www.doi.org/10.1111/biom.13312
- Lähivaara, T., A. Seppänen, J. Kaipio, J. Vauhkonen, L. Korhonen, T. Tokola, and M. Maltamo. 2014. Bayesian approach to tree detection based on airborne laser scanning data. IEEE Transactions on Geoscience and Remote Sensing 52(5):2690-2699.
- Yazigi, A., A. Penttinen, A.K. Ylitalo, M. Maltamo, P. Packalen, and L. Mehtätalo. 2021. Sequential spatial point process models for spatio-temporal point processes: A self-interactive model with application to forest tree data. arXiv preprint.

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