# MODELING HEIGHT-DIAMETER CURVES FOR PREDICTION

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## Introduction

***Height-diameter (H-D) curve*** is one of the first models one needs to fit in a forest inventory. The aims of this study were:

1. To demonstrate the differences between *plot-specific and marginal* and *simple and generalized* H-D curves.
2. Explore the fit of 16 nonlinear functions for the H-D relationship in 28 different datasets of different tree species from different regions.
3. Produce easy-to-use R functions with sensible defaults for tree height imputation.

## The modeling concepts

### *The plot-specific H-D curve*

- Describes the H-D relationship on a sample plot.
- Can be obtained using a mixed-effects model.
- Shown by the lines in Fig. 1a - 1d.

### *The marginal H-D curve*

- Describes the H-D relationship over a population or subpopulation of plots.
- Can be obtained by using a fixed-effect model (Fig. 1e and 1f) or by averaging a plot-specific predictions over the population of plots for each diameter.
- Is often very different from any plot-specific relationship in terms of shape, as illustrated especially by Fig. 1e.

### *The simple model*

- Has only tree diameter as a predictor in the model
- The simple mixed-effect model is sufficient for plot-level prediction using random effects (Fig 1a) but is not sufficient for prediction for plots without calibration measurements (Fig 1c) because the random effects do not have a common mean of 0.
- The simple fixed-effect model could have some use in prediction for a tree selected randomly from the population of trees over a large area.

### *The generalized model*

- Has at least plot-specific mean diameter $\bar{d}_i$ as a predictor in the model, in addition to tree diameter.
- The generalized mixed-effect models is sufficient both for plot-level prediction using random effects (Fig 1b) and for plot-level prediction for plots without calibration measurements (Fig 1d).
- The generalized fixed-effect model could be of use in prediction for a tree selected randomly from a subpopulation of stands with a common mean diameter.

## Results

- The 2-parameter *Näslunds* and *Curtis* functions are suggested for plot-specific H-D relationship.
- See our suggestions on the modeling procedures in Mehtätalo et al. (2015).
- Tools for height imputation are implemented in R-package `lmfor`.

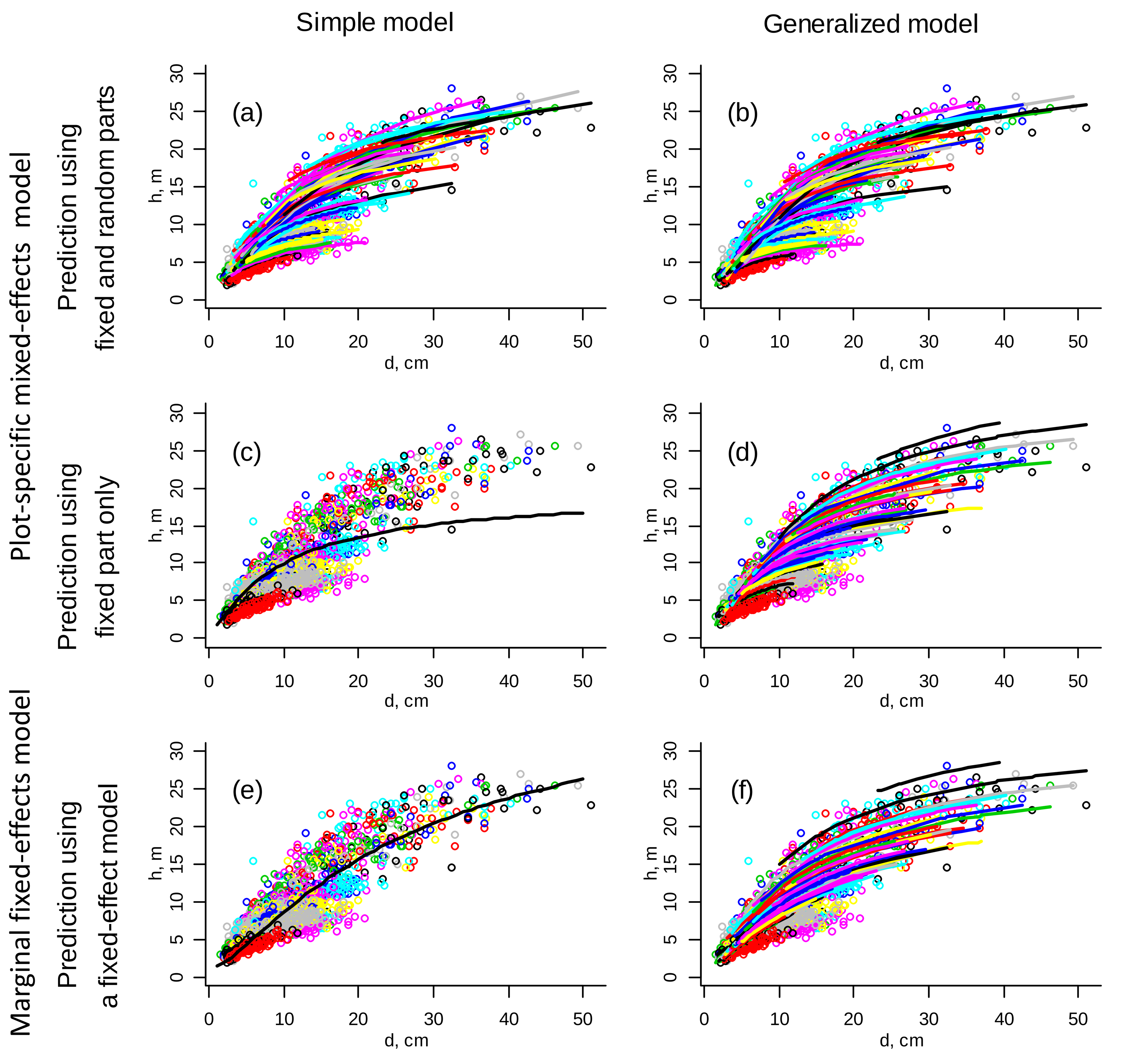


**Figure 1.** Height-Diameter measurements from 56 Scots Pine sample plots in Eastern Finland (dataset `spati` of R-package `lmfor`), overlaid by predictions using simple and generalized mixed-effects and fixed-effect models. The colors indicate the sample plots.

**Table 1.** Example formulations of a simple and generalized H-D model based on the Näslund's function.

| | Simple Näslund's model | Generalized Näslund's model |
|---|---|---|
| Mixed-effects model | $h_{ij} = BH + \frac{d_{ij}^2}{(\alpha_i + \beta_i d_{ij})^2} + e_{ij}$,<br>where<br>$\alpha_i = A + a_i$,<br>$\beta_i = B + b_i$,<br>$\begin{pmatrix} a_i \\ b_i \end{pmatrix} \sim N(0, \boldsymbol{D}), e_{ij} \sim N(0, \sigma_{ij}^2)$,<br>giving<br>$h_{ij} = BH + \frac{d_{ij}^2}{(A + Bd_{ij} + a_i + b_i d_{ij})^2} + e_{ij}$ | $h_{ij} = BH + \frac{d_{ij}^2}{(\alpha_i + \beta_i d_{ij})^2} + e_{ij}$,<br>where<br>$\alpha_i = A_0 + A_1 \bar{d}_i + a_i$,<br>$\beta_i = B_0 + B_1 \bar{d}_i + b_i$,<br>$\begin{pmatrix} a_i \\ b_i \end{pmatrix} \sim N(0, \boldsymbol{D}), e_{ij} \sim N(0, \sigma_{ij}^2)$,<br>giving<br>$h_{ij} = BH + \frac{d_{ij}^2}{(A_0 + A_1 \bar{d}_i + B_0 d_{ij} + B_1 \bar{d}_i d_{ij} + a_i + b_i d_{ij})^2} + e_{ij}$ |
| Fixed-effect model | $h_{ij} = BH + \frac{d_{ij}^2}{(A + Bd_{ij})^2} + e_{ij}$ | $h_{ij} = BH + \frac{d_{ij}^2}{(A_0 + A_1 \bar{d}_i + B_0 d_{ij} + B_1 \bar{d}_i d_{ij})^2} + e_{ij}$ |

## Reference

Mehtätalo, L.. de-Miguel, S, and Gregoire, T.G. 2015. Modeling height-diameter curves for prediction. Canadian Journal of Forest Research, 45(7): 826-837, 10.1139/cjfr-2015-0054.

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