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Recovering plot-specific diameter distribution and Height-Diameter curve using ALS-based stand characteristics

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Background

- Methods have been developed for predicting Stand characteristics (Total volume, Basal area, Mean height, Number of stems) using ALS data.
  - E.g. Naesset 2002.
- In many cases, a more detailed information (E.g. timber assortments) is desired
  - ⇒Information about tree size distribution would be useful

OUR QUESTION

- How to obtain a sufficiently detailed **stand description** that is compatible with predictions of volume (V), number of stems (N), mean diameter (D) and mean height (H)?

Stand description

- Our description includes
  - Stand density (# of stems)
  - Diameter distribution
  - H-D curve
- The growing stock of a single-species stand is described with four parameters:
  - Number of stems ($\hat{N}$)
  - a scale parameter of H-D curve (γ)
  - scale (α) and shape (β) of the assumed diameter distribution (Weibull)

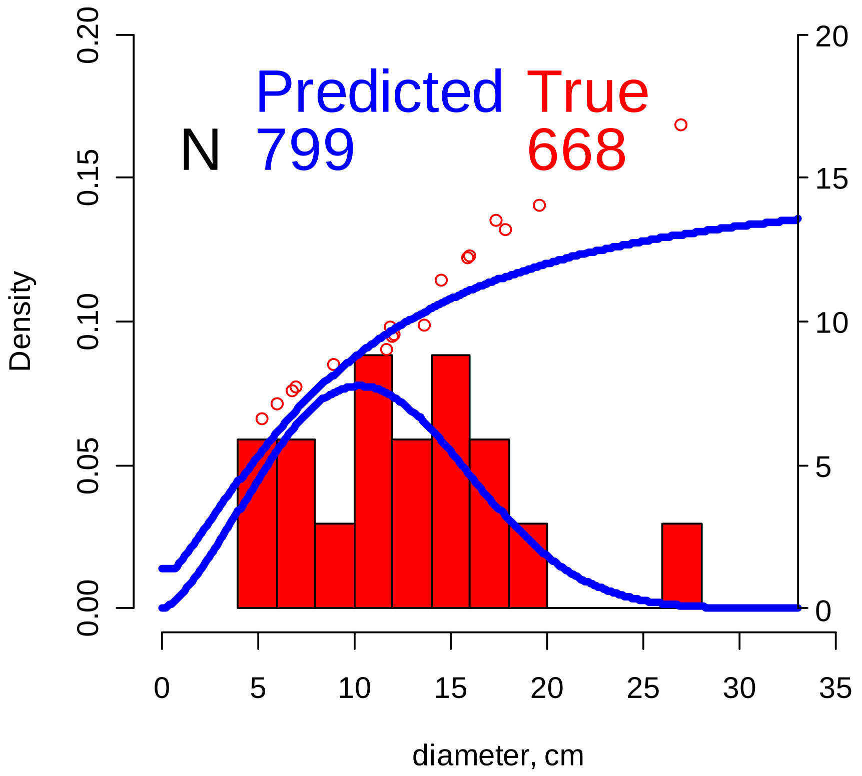


Fig 1. A theoretical stand description (blue) and the actual growing stock (red).

Aims of the study

- To develop a **Parameter Recovery Method (PRM)** for obtaining a compatible stand description using laser data
- Test the method
  - Develop prediction models for Volume, number of stems, mean diameter and mean height
  - Use the models for prediction in the modeling data
  - Formulate **compatible** diameter distribution and Height-diameter curve for each stand
  - Evaluate the accuracy of these stand descriptions
    - by calculating volumes above different diameter limits
    - by plotting the obtainend stand descriptions
    - by studying the feasibility of the approach

$\hat{V}$

Idea of parameter recovery

- Set the quantities (*N*, *H*, *D* and *V*) based on ALS data equal to those based on our stand description:

$$\begin{cases} V(\boldsymbol{\theta}, \hat{N}) - \hat{V} = 0 \\ D(\boldsymbol{\theta}) - \hat{D} = 0 \\ H(\boldsymbol{\theta}) - \hat{H} = 0 \end{cases}$$

where the values with hat are the predictions based on ALS data and *V*(), *H*() and *D*() give volume, mean height and mean diameter as a function of $\boldsymbol{\theta} = (\alpha, \beta, \gamma)'$

- Solve for $\boldsymbol{\theta} = (\alpha, \beta, \gamma)'$
- Generalizes the Parameter Recovery Method (PRM) for diameter distribution to PRM for a stand description.
- Solution does not necessarily exsist!

V, D and H as a function of θ

$$V\left(\boldsymbol{\theta}, \hat{N}\right) = \hat{N}\int_0^{\infty} f\left(x \middle| \alpha, \beta\right) v\left(x, h\left(x \middle| \gamma\right)\right) dx$$

$$D\left(\boldsymbol{\theta}\right) = F_G^{-1}\left(0.5 \middle| \alpha, \beta\right)$$

$$H\left(\boldsymbol{\theta}\right) = h\left(\hat{D} \middle| \gamma\right),$$

Note

$$F_G\left(x \middle| \alpha, \beta\right) = \int_0^{x} f_G\left(u \middle| \alpha, \beta\right) du\ ,$$

$$f_G\left(x \middle| \alpha, \beta\right) = \frac{x^2 f\left(x \middle| \alpha, \beta\right)}{\int_0^{\infty} x^2 f\left(x \middle| \alpha, \beta\right) dx}$$

$\hat{N}$ is stand density,

x is tree diameter,

$h\left(x \middle| \gamma\right)$ is height for diameter *x*, (Mehtätalo 2005)

$f\left(x \middle| \alpha, \beta\right)$ is density of diameter distribution (2-parameter Weibull)

$v(x,h)$ is volume for tree with diameter *x* and height *h*, (Laasasenaho 1982) and

$\boldsymbol{\theta} = \left(\alpha, \beta, \gamma\right)$ includes the parameters of stand description (α and β for diameter distribution, γ for H-D –curve).

Solving the system of equations

- Starting from $\boldsymbol{\theta} = \left(5, \hat{D}, 3\right)$, we first minimized the sum of squared differences

$$\left(V\left(\alpha,\beta,\gamma,\hat{N}\right)-\hat{V}\right)^2+\left(D(\alpha,\beta)-\hat{D}\right)^2+\left(H(\gamma)-\hat{H}\right)^2$$

for $\boldsymbol{\theta}$ using the Nelder-Mead algorithm as implemented in R function `optim`

- The final estimates were then solved using Newton-Raphson algorithm
- Numerical methods were used in approximating the integrals that could not be analytically solved
    - (R function `integrate`).

Data

- 506 circular sample plots were measured collected from Juuka area during summers 2005 and 2006
- Laser scanning in July, 2005 (0.6 pulses per square meter)
- We used 213 pure Scots pine plots (90% or more of pine)

	mean	min	max	sd
Volume, m^3ha^{-1}	122.8	14.7	317.8	61.8
Number of stems, ha^{-1}	903.8	196	2122	377.3
Basal area median diameter, cm	18.1	9.4	40.0	4.8
Height of a basal area median tree, m	14.0	6.0	23.4	3.3

Table 1. Mean characteristics of the study data. Sd is standard deviation.

Models for V, N, H and D

$$\ln V = 0.134 + 1.202\ln\left(f\_h_{50}\right) + 0.198\sqrt{f\_veg} + 0.114\ln\left(l\_veg\right)$$

$$\ln N = 7.803 - 1.027\ln\left(f\_h_{95}\right) + 0.251\sqrt{f\_veg} + 7.988\left(\frac{1}{f\_i_{50}}\right) - 0.319\ln\left(l\_p_{20}\right)$$

$$\ln H = -26.075 + 5.747\ln f\_h_{95} + 3.581\ln f\_h_{40} + 38.371\frac{1}{f\_h_{60}} + 0.605\ln l\_veg + 4.907\ln l\_h_{50}$$

$$\ln D = 2.697 - 2.605\frac{1}{f\_i_{50}} - 37.812\frac{1}{f\_p_{20}} + 2.004\ln f\_h_{50} - 1.231\ln f\_h_{40}$$

f and *l* denotes pulse type (first or last),

h_p the pth percentile of laser height distribution,

veg is the proportion of vegetation hits,

i_{50} the 50th percentile of intensity reflection, and

p_{20} the proportion of laser hits accumulated at the height of 20%.

R-square ranged from 0.5 for ln*N* to 0.92 for ln*V*

Performance in modeling data

	RMSE		Bias	
	Absolute	%	Absolute	%
H, m	1.22	8.70	0.00	-0.01
D, cm	2.35	12.96	-0.15	-0.80
N, ha^{-1}	279.8	31.00	-35.30	-3.91
V, m^3ha^{-1}	20.02	16.29	-1.64	-1.33

Table 2. RMSE and bias of predicted stand characteristics in the data of feasible solutions (211 plots).

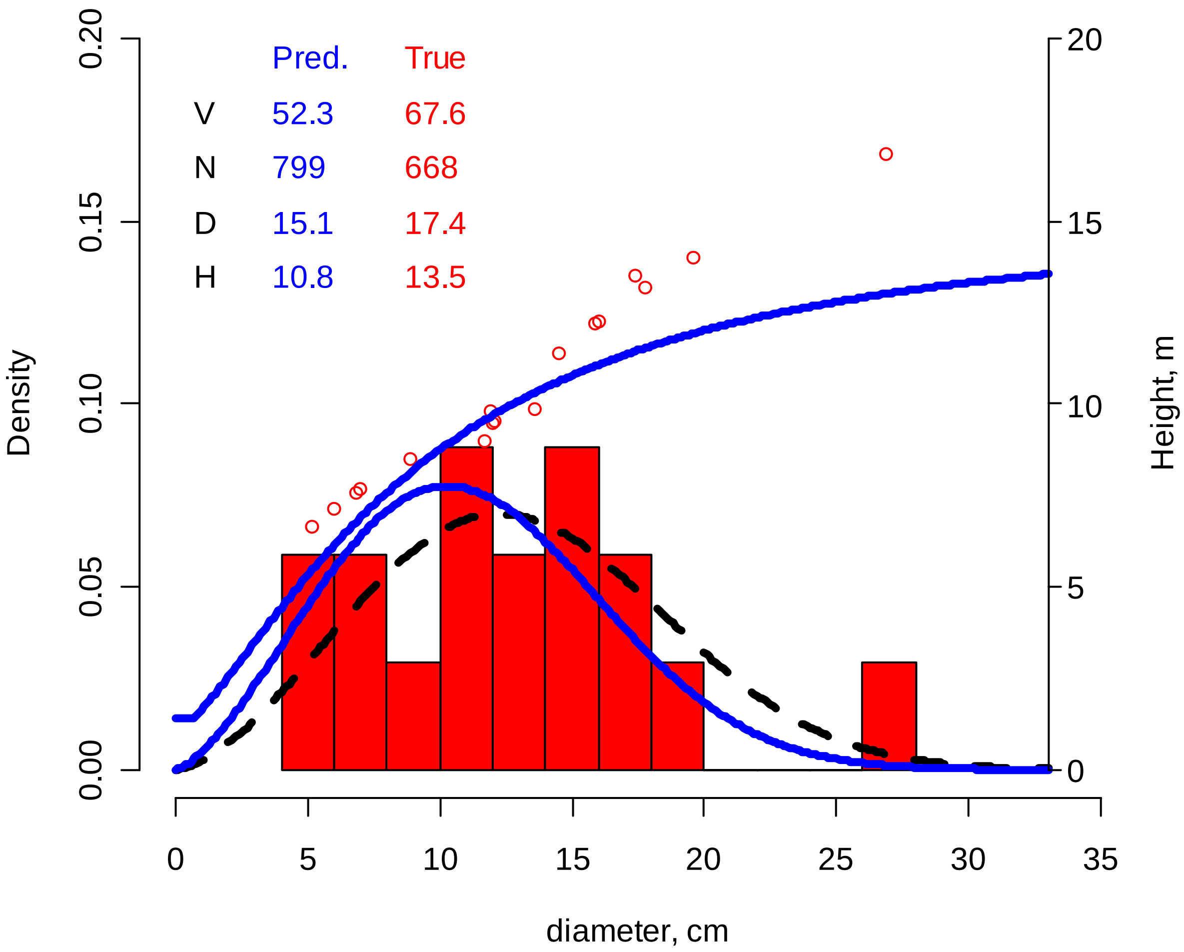
Performance of the recovery in modeling data

	RMSE		Bias	
	Absolute	%	Absolute	%
V_{10}, m^3 ha^{-1}	19.70	16.67	-0.79	-0.67
V_{15}, m^3 ha^{-1}	22.20	22.70	-2.20	-2.25
V_{20}, m^3 ha^{-1}	24.33	42.76	-1.72	-3.02

Table 3. RMSE and bias of volumes above 10, 15 and 20 cm diameter limit in the data of feasible solutions (211 plots).

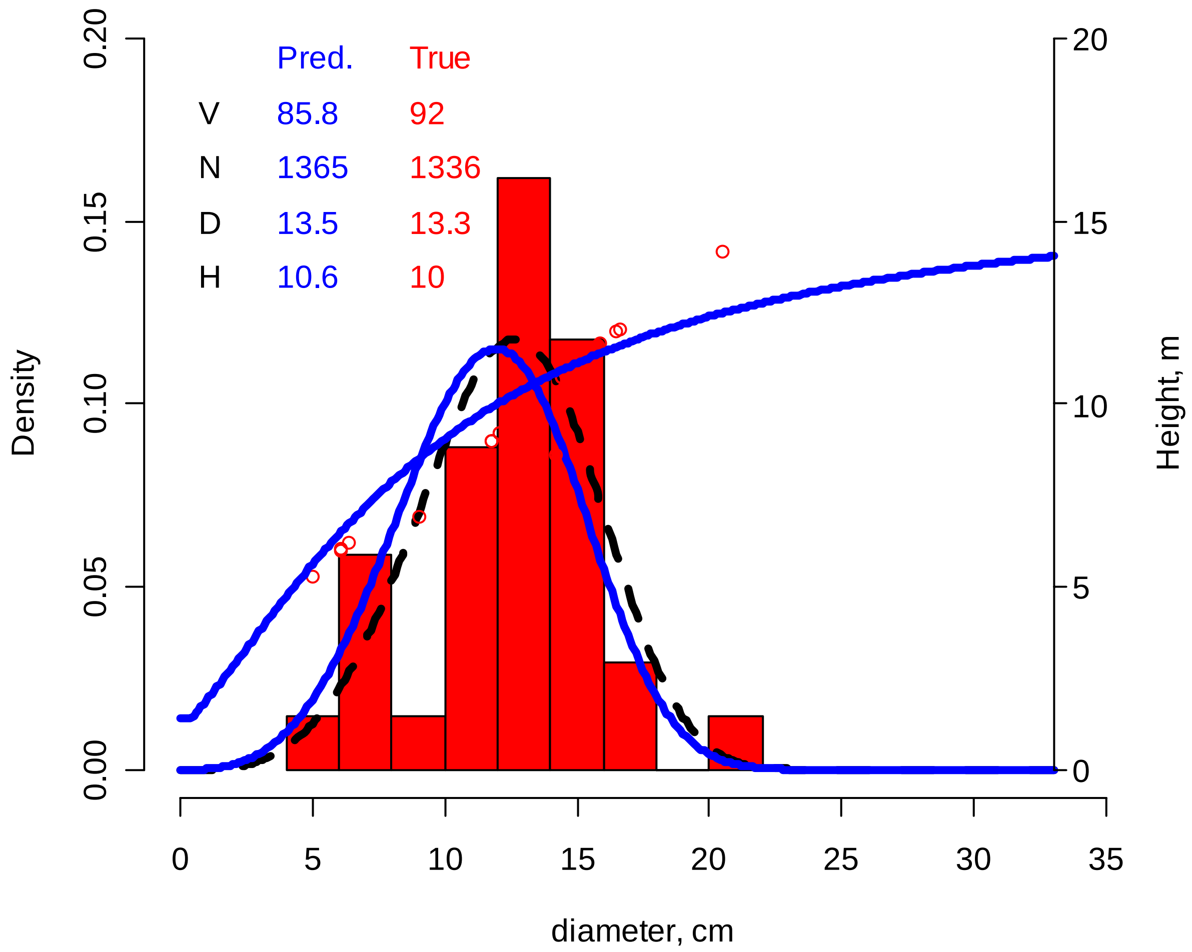
Infeasible in two plots!

Example 1

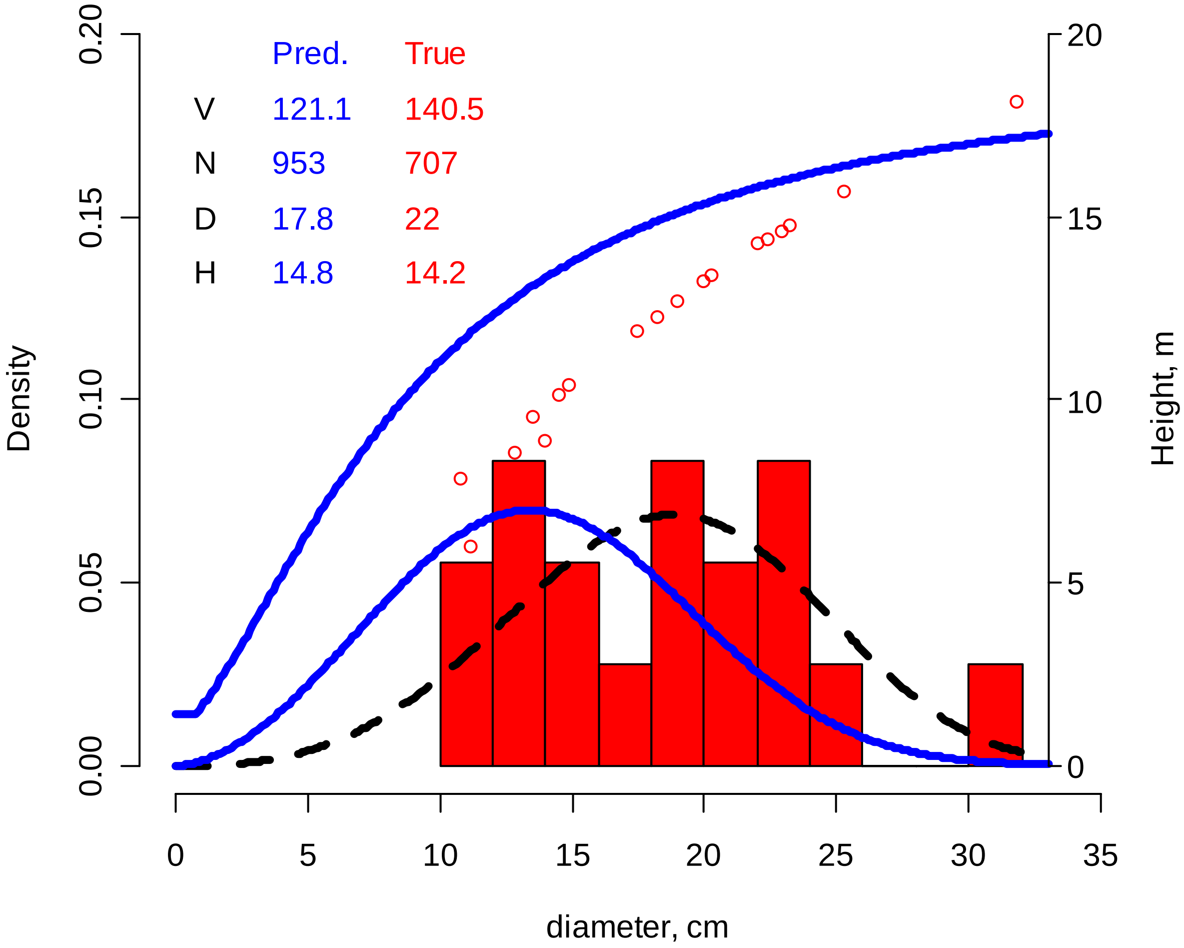


Pred.
True
V
52.3
67.6
N
799
668
D
15.1
17.4
H
10.8
13.5
Density
Height, m
diameter, cm
0.00
0.05
0.10
0.15
0.20
0
5
10
15
20
25
30
35

Example 2


Pred.
True
V 85.8 92
N 1365 1336
D 13.5 13.3
H 10.6 10
Density
Height, m
diameter, cm
0.00
0.05
0.10
0.15
0.20
0
5
10
15
20
0
5
10
15
20
25
30
35

Example 3



Pred.
True
V 121.1 140.5
N 953 707
D 17.8 22
H 14.8 14.2
Density
0.00
0.05
0.10
0.15
0.20
Height, m
0
5
10
15
20
0 5 10 15 20 25 30 35
diameter, cm

Discussion

- Errors in mean diameter move the whole distribution -> inaccuracy or even infeasibility
- Generalizes the idea of PRM from diameter distributions to a stand description including diameter distribution and H-D curve.
- Could be used with other sets of four stand variables
- Infeasibility is a problem
  - Could the initial solution be used?
  - A more sophisticated Stand description would require more variables to be predicted
- Stand structure could be predicted also for old inventory data
- We used pure scots pine stands, which should be known *a priori*

Thank you!

Questions?