Recovering forest parameters from canopy height observations

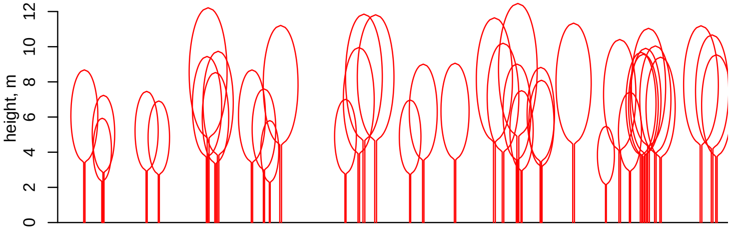
Lauri Mehtätalo[1] and Jukka Nyblom[2]

[1] FFRI, Joensuu, Finland and Yale University
[2] University of Jyväskylä, Finland

October 31, 2006
10th Annual NEMO Meeting
Grey Towers, Milford, PA

Outline of the presentation

Forest inventory using laser scanning



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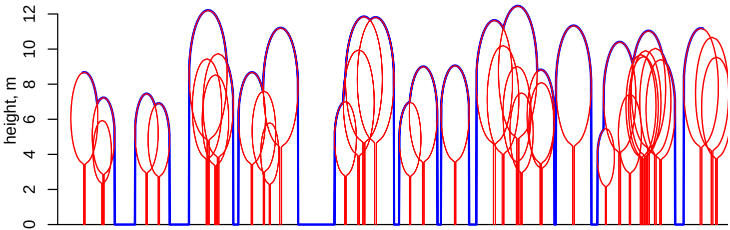

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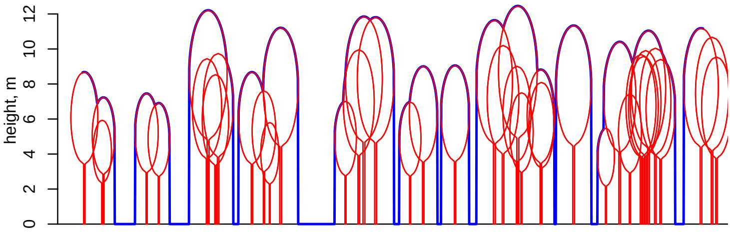
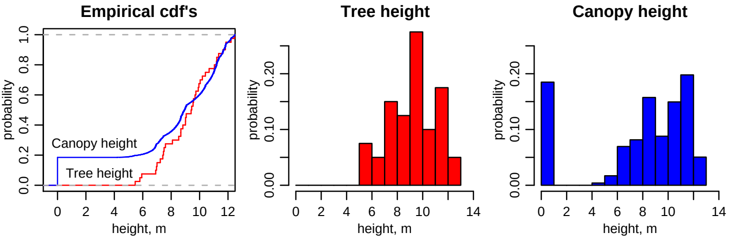


- In forest inventory, we are interested in the number, species and size of the trees.
- From above we can see only the surface of the forest stand.
- In airborne laser scanning, we essentially measure height of this surface at given points, i.e. the distribution of canopy height.

Canopy height (CH) vs tree height (TH)



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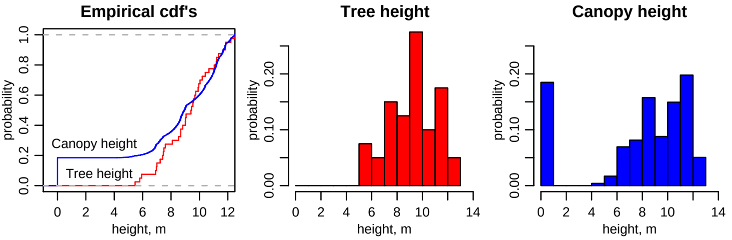

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What kind of stand most likely produced the distribution of canopy heights we observed?

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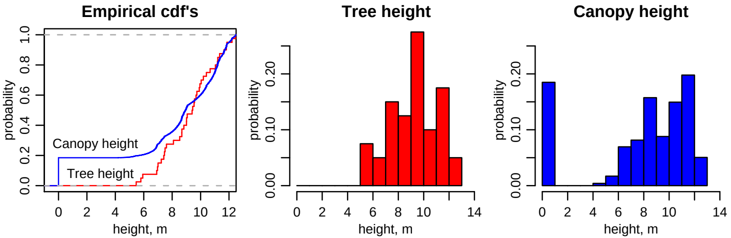

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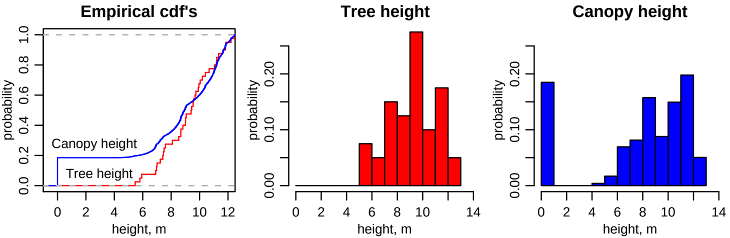

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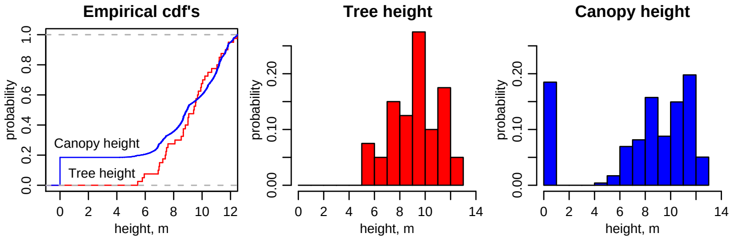

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- The essential task is to state the distribution of canopy heights in terms of the distribution of tree heights, stand density and species composition.

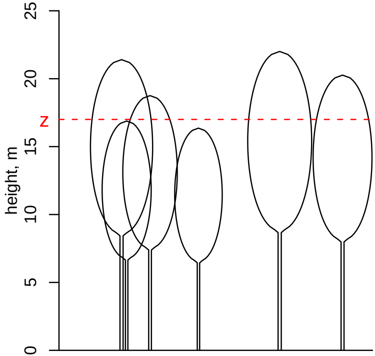
Then our question can be answered by fitting the recovered distribution to the observed data using Maximum Likelihood.

Distribution of canopy heights

Let Z be canopy height at a random point v, and z a fixed reference height.

The distribution of canopy heights is

$$F(z) = P(Z \leq z)$$



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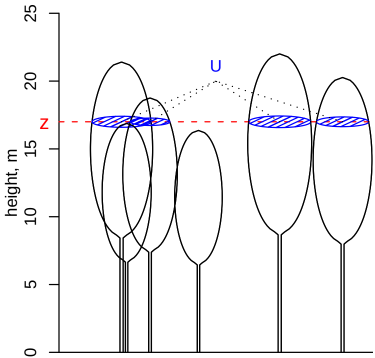

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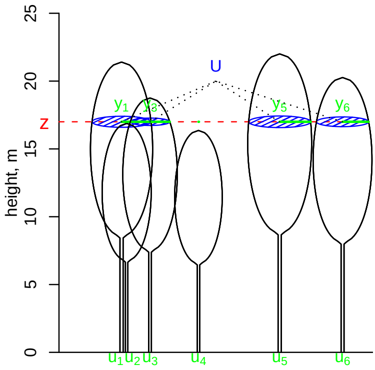
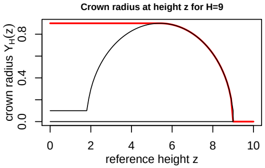


The effect of crown shape

Crown shape links the cross-sectional crown area with tree height. Assume that tree heights H_i are i.i.d random variable with density, $f_H(h|\boldsymbol{\theta})$.
Denote tree locations by u_i, $i = 1, \ldots, N$.

For tree i, we define the crown radius at height z as $Y(z, H_i)$. This function is assumed to be known.



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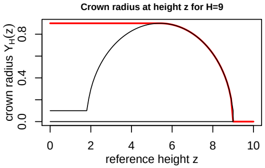
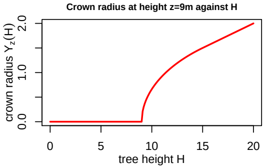


Fixing the reference height z,
$Y(z, H_i) = Y_z(H_i)$ gives the crown radius of tree i with height H_i.

Thus, $Y_z(H_i)$ is a random variable with distribution $F_H(Y_z^{-1}(y)|\boldsymbol{\theta})$.



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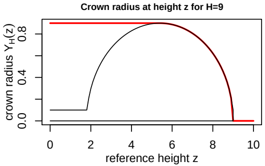

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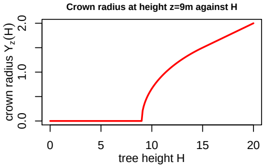
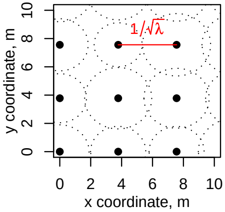


We get $F(z) = \prod_{i=1}^{N} P(\|v - u_i\| > Y_i(z)) = \prod_{i=1}^{N} F_H(Y_z^{-1}(\|v - u_i\|))$

Single-species forest, regular tree locations

Assume that λ trees per m^2 are located at the nodes of a square grid.



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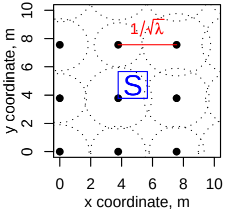

The c.d.f and density of Z become

$$F_Z(z|\lambda,\boldsymbol{\theta}) = \begin{cases} 4\lambda \int_{v\in S} \prod_{i=i}^{N} F_H\left[Y_z^{-1}(\|u_i - v\|)|\boldsymbol{\theta}\right]\mathrm{d}v & z \geq 0 \\ 0 & z < 0 \end{cases},$$

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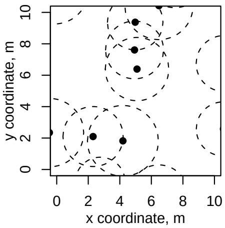


The c.d.f and density of Z become

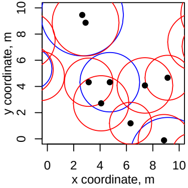
$$F_Z(z|\lambda, \boldsymbol{\theta}) = \begin{cases} e^{-\lambda \pi E\left[Y(z,H_i)^2\right]} & z \geq 0 \\ 0 & z < 0 \end{cases}$$

$$f_Z(z|\lambda, \boldsymbol{\theta}) = \begin{cases} -\lambda\pi F_Z(z|\lambda, \boldsymbol{\theta}) \int_z^{b^{-1}(z)} \frac{\mathrm{d}}{\mathrm{d}z}[Y(z,h)^2] f_H(h) \mathrm{d}h & z > 0 \\ F_Z(z|\lambda, \boldsymbol{\theta}) & z = 0 \\ 0 & z < 0 \end{cases},$$

where $b(h)$ gives the height of maximum crown radius as a function of tree height.

Two-species Poisson forest

Assume a mixed stand with density λ and proportions ρ and $1-\rho$ for species 1 and 2. The crown shape and distribution of tree height for species 1 are $Y_1(z, h)$ and $f_1(h|\boldsymbol{\theta}_1)$ and correspondingly for species 2.



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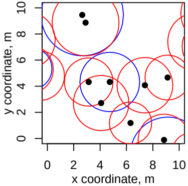

The c.d.f. and density of Z become

$$F_Z(z|\lambda,\rho,\boldsymbol{\theta}) = \begin{cases} \exp\Big[-\lambda\pi\big[\rho\int_0^\infty [Y_1(z,h)^2] f_1(h|\boldsymbol{\theta}_1)\mathrm{d}h \\ \qquad +(1-\rho)\int_0^\infty [Y_2(z,h)^2] f_2(h|\boldsymbol{\theta}_2)\mathrm{d}h\big]\Big] & z \geq 0 \\ F_Z(z|\lambda,\boldsymbol{\theta}) = 0 & z < 0 \end{cases}$$

$$f_Z(z|\lambda,\rho,\boldsymbol{\theta}) = \begin{cases} -\lambda\pi F_Z(z|\lambda,\rho,\boldsymbol{\theta})\big[\rho I_1(z|\boldsymbol{\theta}_1) + (1-\rho) I_1(z|\boldsymbol{\theta}_2)\big] & z > 0 \\ F_Z(z|\lambda,\rho,\boldsymbol{\theta}) & z = 0 \\ 0 & z < 0 \end{cases},$$

where

$$I_j(z|\boldsymbol{\theta}_\mathbf{j}) = \int_z^{b_j^{-1}(z)} \frac{\mathrm{d}}{\mathrm{d}z}[Y_j(z,h)^2] f_j(h|\boldsymbol{\theta}_j)\mathrm{d}h\,.$$

Estimation

- We first solve $\lambda(\mathit{canopy\ closure}, \boldsymbol{\theta}, [\rho])$ from

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- The ML estimates of $\boldsymbol{\theta}$ (and ρ) are found by minimizing

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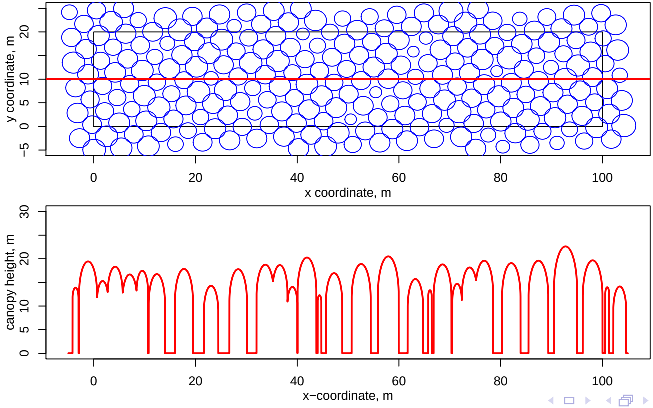
- The value of λ is then solved from the above equation

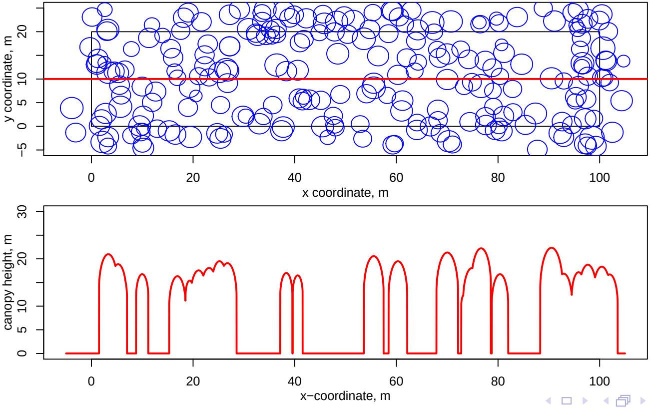
Stand 1, 'plantation'

- Trees located in a square grid
- 700 trees per ha
- Height ∼Weibull(10,20)
- Crowns are ellipsoids with half axes $0.4h$ and $0.1h$



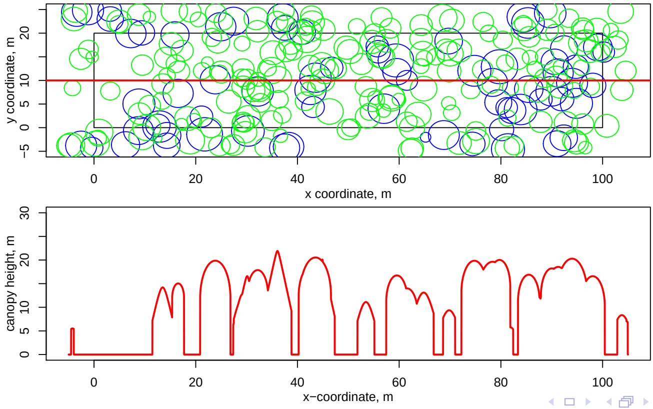
Stand 2, ‘natural’

- Trees located randomly
- 700 trees per ha
- Height ∼Weibull(10,20)
- Crowns are ellipsoids with half axes $0.4h$ and $0.1h$



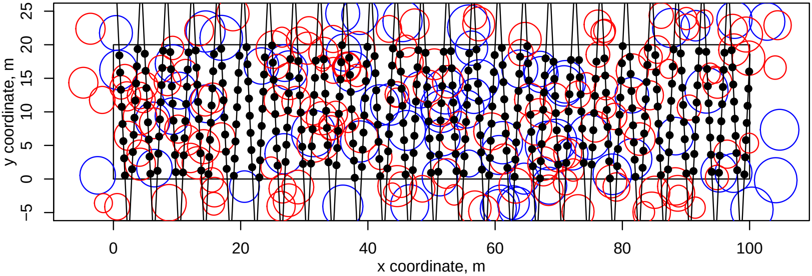
Stand 3 'mixed'

- Trees located randomly
- 200 ellipsoid shaped and 500 conical shaped trees per ha
- Ellipsoid: Height∼Weibull(9,20), Conical: Height∼Weibull(7,19)
- Ellipsoid half axes 0.4h and 0.15h, cone height 0.6h and width 0.24h



Taking observations

- Five 100*20 m^2 sample plots from each stand
- Total of 384–394 canopy height observations per plot



Stand 1, 'plantation'

Table: Estimation results of Stand 1. True values were $\alpha = 10$, $\beta = 20$, 10000λ=700 trees per ha, $\overline{H}$=19.03 m.

	Estimates					Sample values	
Plot	$\widehat{cc}$	$\widehat{\alpha}$	$\widehat{\beta}$	$10000\widehat{\lambda}$	$\widehat{\overline{H}}$	10000λ	$\overline{H}$
1	0.824	10.76	20.19	730	19.27	705	19.30
2	0.803	9.91	19.83	735	18.86	700	18.93
3	0.775	10.00	20.13	679	19.16	705	18.97
4	0.797	9.00	19.60	752	18.56	710	18.88
5	0.795	9.98	19.95	715	18.98	690	18.97

- Point estimates accurate, λ and mean H follow sample values

Stand 2, 'natural'

Table: Estimation results of Stand 2. True values were $\alpha = 10$, $\beta = 20$, 10000λ=700 trees per ha, $\overline{H}$=19.03 m.

	Estimates					Sample values	
Plot	$\widehat{cc}$	$\widehat{\alpha}$	$\widehat{\beta}$	$10000\widehat{\lambda}$	$\widehat{\overline{H}}$	10000λ	$\overline{H}$
1	0.522	9.69	19.99	641	19.00	625	18.93
2	0.574	10.07	20.11	731	19.13	775	18.90
3	0.550	9.36	19.76	711	18.75	730	18.98
4	0.567	9.40	19.80	744	18.78	780	18.66
5	0.536	9.86	20.00	667	19.01	645	19.14

- Point estimates less accurate than in stand 1 but good

Stand 3, 'mixed'

Table: Estimation results of the mixed stand. True values were $\alpha_1 = 9$, $\beta_1 = 20$, $\alpha_2 = 7$, $\beta_2 = 19$, $\rho = 0.286$, $10000\lambda_1$=200 trees per ha, $10000\lambda_2$=500 trees per ha, $\overline{H}_1$=18.94 m, $\overline{H}_2$=17.77 m

	Estimates					
Plot	cc	$\widehat{\alpha_1}$	$\widehat{\beta_1}$	$\widehat{\alpha_2}$	$\widehat{\beta_2}$	$\widehat{\rho}$
1	0.681	9.25	20.00	12.85	17.53	0.303
2	0.723	12.70	21.07	6.22	19.42	0.210
3	0.712	9.11	19.80	9.46	20.20	0.291
4	0.661	11.48	20.23	7.36	21.04	0.268
5	0.733	8.47	19.84	7.88	17.44	0.273

	Estimates				Sample values			
Plot	$10000\widehat{\lambda_1}$	$10000\widehat{\lambda_2}$	$\widehat{\overline{H}_1}$	$\widehat{\overline{H}_2}$	$10000\lambda_1$	$10000\lambda_2$	$\overline{H}_1$	$\overline{H}_2$
1	206	472	18.96	16.84	175	445	19.41	17.92
2	148	558	20.23	18.05	185	570	19.03	17.77
3	188	457	18.76	19.17	180	495	19.18	18.06
4	142	389	19.35	19.73	220	445	18.68	17.84
5	226	602	18.73	16.41	195	590	18.65	17.38

- Estimates good, α_2 the least accurate

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- No need for dense data, actually works better with less dense data
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- Capability to "species recognition" according to crown shape
- Estimated stand structure is obtained
- Codominant or undergrowth trees should not be a problem

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- How to speed up computation?
- How to find initial guesses of estimates?

Publications

Mehtätalo, L. and Nyblom, J. Retrieving forest parameters from observations of canopy height. Manuscript.

Mehtätalo, L. 2006. Eliminating the effect of overlapping crowns from aerial inventory estimates. Canadian Journal of Forest Research 36(7): 1649-1660. (Reprints available upon request)