Application of mixed-effect model predictions in forestry

Lauri Mehtätalo[1]

[1] University of Eastern Finland, School of Computing & School of Forest Sciences

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Types of forest datasets

- Forest datasets are usually hierarchical e.g.
  - needles within branches
  - branches within trees
  - **trees within sample plots**
  - sample plots within forest stands
  - forest stand within regions
  - repeated measurements of trees, branches etc.
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These datasets are naturally modeled using random effect models.

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- Using mixed-effects models with hierarchical datasets result in
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  2. Possibilitý to compute the predictions at different levels of the dataset.
- If the main interest is the inference (e.g. the effects of certain medical treatments on individuals) the first property is more important.
- If the main interest is prediction, then greatest benefit may arise from the possibility to make predictions at different levels of hierarchy. **This is possible also for observations from outside the modeling data**

Topic of this presentation

I will demonstrate and discuss the use of mixed-effects models in four forestry situations (The main benefit of mixed-effects models arsing either from prediction (P) or inference (I)).

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- Using nonlinear mixed-effect-models to analyse the previously extracted tree-level thinning effects (I)
- Using a system of linear mixed-effects models to predict of tree level size and quality-related characteristics using remotely sensed information and field sample tree measurements (P)

Simple mixed-effects models

Let y_{ki} be the observed response for individual i in group k, and let x_{ki} be a fixed predictor, just as in the one-predictor regression. In a linear mixed-effects model, one may have both fixed (population level) parameters and random parameters, e.g.,

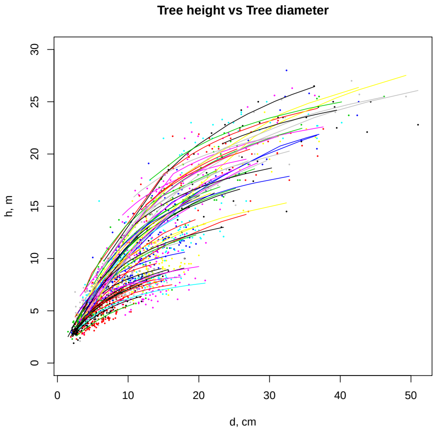
$$y_{ki} = a + bx_{ki} + \alpha_k + \beta_k x_{ki} + \epsilon_{ki} ,$$

where $(\alpha_k, \beta_k)' \sim MVN(0, \boldsymbol{D})$ and $\epsilon_{ki} \sim N(0, \sigma^2)$. a and b are the fixed parameters.

- The model allows population and group-level predictions: $\widehat{a} + \widehat{b}x_{ki}$ and $\widehat{a} + \widehat{\alpha_k} + (\widehat{b} + \widehat{\beta}_k)x_{ki}$ and corresponding residuals
- Generalizes to multiple groupings, with either nested or crossed grouring structures
- Generalizes to nonlinear models, where some parameters of a nonlinear function are allowed to vary between groups

Why an H-D model?

- H-D relationship varies much among sample plots, but height measurement is time-consuming.



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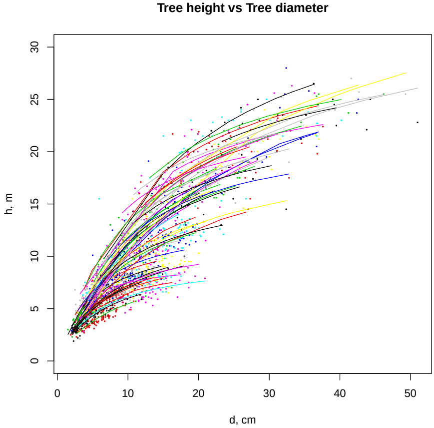

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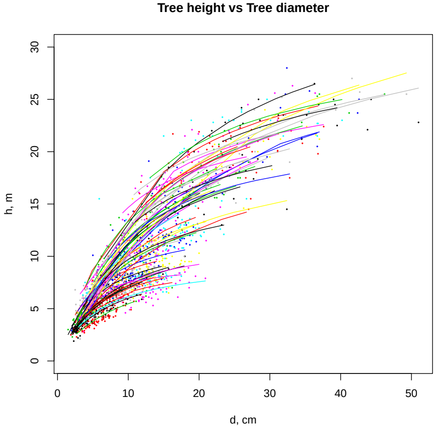


If a previously fitted H-D model is available, it can be localized, or calibrated, for the new plot by predicting the random effects using the sampled tree heights.

The Height-Diameter model

The logarithmic heigth H_{kti} for tree i in stand k at time t with diameter D_{kti} at the breast height is expressed by

$$\ln(H_{kti}) = a(DGM_{kt}) + \alpha_k + \alpha_{kt} + (b(DGM_{kt}) + \beta_k + \beta_{kt})D_{kti} + \epsilon_{kti} ,$$

where $a(DGM_{kt})$ and $b(DGM_{kt})$ are known fixed functions of plot-specific mean diameter DGM_{kt}, $(\alpha_k, \beta_k)'$ and $(\alpha_{kt}, \beta_{kt})'$ are the plot and measurement occasion -level random effects with varainces (correlations)

$$\mathrm{var}\begin{bmatrix}\alpha_k \\ \beta_k\end{bmatrix} = \begin{bmatrix} 0.108^2 & (0.269) \\ 0.0028 & 0.0958^2 \end{bmatrix} \qquad \mathrm{var}\begin{bmatrix}\alpha_{kt} \\ \beta_{kt}\end{bmatrix} = \begin{bmatrix} 0.0168^2 & (-0.681) \\ -0.0003 & 0.0223^2 \end{bmatrix}$$

and ϵ_{kti} are independent normal residuals with $\mathrm{var}(\epsilon_{kti}) = 0.401^2 \left(\max(D_{kti}, 7.5)\right)^{-1.068}$

The stand level mixed-effects model

The sample tree heights of a new stand can be described by

$$\boldsymbol{y} = \boldsymbol{\mu} + \boldsymbol{Z}\boldsymbol{b} + \boldsymbol{\epsilon}\,,$$

where
$\boldsymbol{y}$ includes the observed sample tree heights,
$\boldsymbol{\mu}$ is the fixed part,
$\boldsymbol{b} = \left(\alpha_k \quad \beta_k \quad \alpha_{k1} \quad \beta_{k1} \quad \alpha_{k2} \quad \beta_{k2} \quad \ldots \right)'$ includes the random effects,
$\boldsymbol{Z}$ is the corresponding design matrix, and
$\boldsymbol{\epsilon}$ includes the residuals.
We denote $\mathrm{var}(\boldsymbol{b}) = \boldsymbol{D}$ and $\mathrm{var}(\boldsymbol{\epsilon}) = \boldsymbol{R}$.

Prediction of random effects

The variances and covariances between random effects and observed heights can be written as

$$\begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{y} \end{bmatrix} \sim \left(\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{\mu} \end{bmatrix}, \begin{bmatrix} \boldsymbol{D} & \boldsymbol{DZ'} \\ \boldsymbol{ZD} & \boldsymbol{ZDZ'} + \boldsymbol{R} \end{bmatrix} \right)$$

The Empirical Best Linear Unbiased Predictor (EBLUP) of random effects is

$$\widehat{\boldsymbol{b}} = \boldsymbol{DZ}'(\boldsymbol{ZDZ'} + \boldsymbol{R})^{-1}(\boldsymbol{y} - \boldsymbol{\mu}) .$$

and the variance of prediction errors is

$$\mathrm{var}(\widehat{\boldsymbol{b}} - \boldsymbol{b}) = \boldsymbol{D} - \boldsymbol{DZ}'(\boldsymbol{ZDZ}' + \boldsymbol{R})^{-1}\boldsymbol{ZD}$$

Example

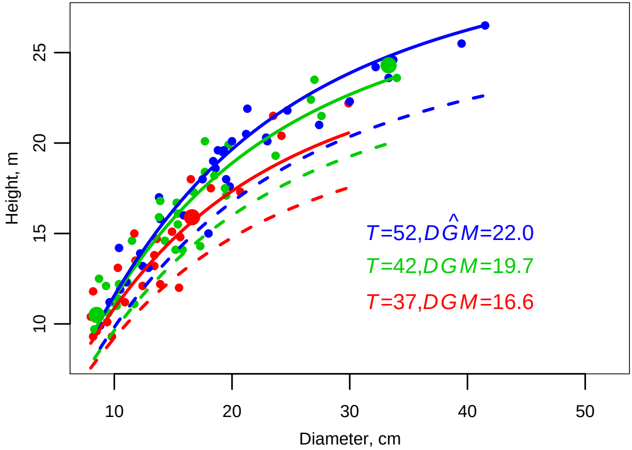
Height of one tree was measured 5 years ago and 2 trees at the current year. The matrices and vectors are

$$\boldsymbol{\mu} = \begin{bmatrix} 2.59 \\ 2.11 \\ 2.99 \end{bmatrix} \quad \boldsymbol{y} = \begin{bmatrix} 2.77 \\ 2.35 \\ 3.19 \end{bmatrix}$$

$$\boldsymbol{Z} = \begin{bmatrix} 1 & -0.36 & 1 & -0.36 & 0 & 0 \\ 1 & -1.22 & 0 & 0 & 1 & -1.22 \\ 1 & 0.058 & 0 & 0 & 1 & 0.058 \end{bmatrix} \quad \boldsymbol{R} = \begin{bmatrix} 0.008 & 0 & 0 \\ 0 & 0.016 & 0 \\ 0 & 0 & 0.004 \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} \alpha_k \\ \beta_k \\ \alpha_{k1} \\ \beta_{k1} \\ \alpha_{k2} \\ \beta_{k2} \end{bmatrix} \quad \boldsymbol{D} = \begin{bmatrix} 0.0118 & 0.0028 & 0 & 0 & 0 & 0 \\ 0.0028 & 0.0092 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0003 & 0.0004 & 0 & 0 \\ 0 & 0 & 0.0004 & 0.0005 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0003 & 0.0004 \\ 0 & 0 & 0 & 0 & 0.0004 & 0.0005 \end{bmatrix}$$

Uncalibrated and calibrated predictions



dashed=fixed part only; solid= calibrated (fixed+random)

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- To understand the dynamics of thinning, one may wish to analyse the effect of thinnings on tree growth.
- However, the growth is affected also by other factors, especially by the site productivity, tree age, and annual weather.
- Mixed-effects models can be used to model out these nuisance effects.

Study material

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- 88 trees were felled in 2006, and the complete time series of diameter increments between 1983 and 2006 was measured for each tree using an X-ray densiometer.

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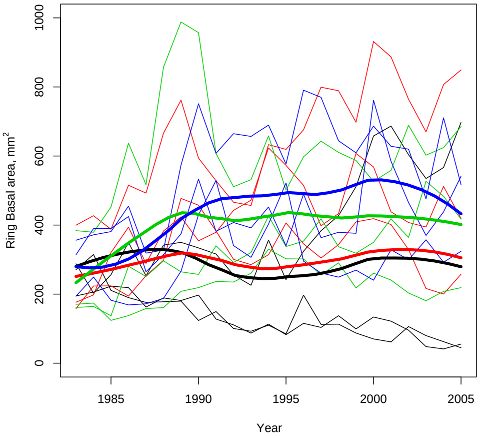
- 88 trees were felled in 2006, and the complete time series of diameter increments between 1983 and 2006 was measured for each tree using an X-ray densiometer.
- The diameter growths were transformed to basal area growths, because $Volume \sim Diameter^2 Height$)

The raw data



- THICK: treatment-specific trends
- THIN: 12 randomly selected trees
- One can see
  - (Age trend)
  - climate-related year effects
  - tree effects

I (control) - black; II (light) - red
III (moderate) - green; IV (heavy) - blue

Modeling the non-thinned response

- A dataset without thinning treatments was produced by including from the original data
  - The control treatment for whole follow-up period
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- A dataset without thinning treatments was produced by including from the original data
  - The control treatment for whole follow-up period
  - The thinned treatments until the year of thinning (1986)
- A linear mixed effect model with random year and tree effects was fitted to the unthinned data

$$y_{kt} = f(T_{kt}) + u_k + v_t + e_{kt} \quad (1)$$

where y_{ckt} is the basal area growth of tree k at year t,
$f(T_{ckt})$ is the age trend (modeled using a spline),
u_k is a NID tree effect,
v_t is a NID year effect and
e_{kt} is a NID residual.

Extracting the thinning effects

- Using the estimated age trend and predicted year and tree effects, the growth without thinning, $\tilde{y}_{kt}$ was predicted for treatments II -IV after the thinning year.

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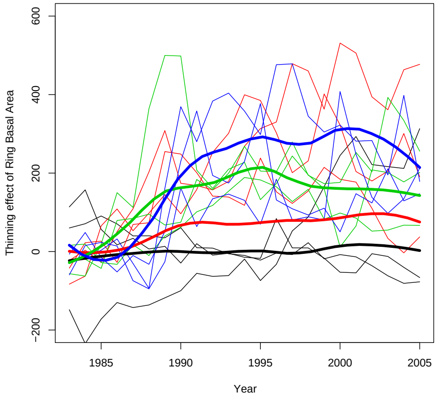
- Using the estimated age trend and predicted year and tree effects, the growth without thinning, $\tilde{y}_{kt}$ was predicted for treatments II -IV after the thinning year.
- The pure thinning effects were estimated by subtracting the prediction from the observed growth

$$d_{kt} = y_{kt} - \tilde{y}_{kt} \quad (2)$$

The estimated thinning effects

Extracted thinning effects



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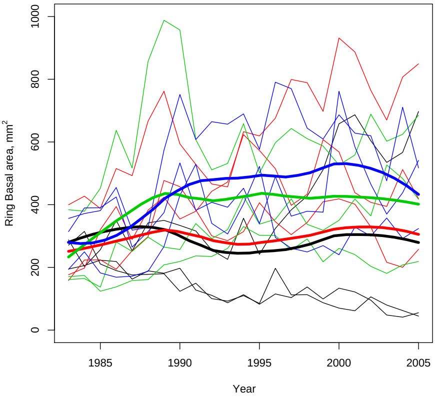


Line color specifies treatment (I:black, II: red, III: green IV: blue). Thick lines show the treatment-specific mean trends; thin lines show 12 randomly selected trees.

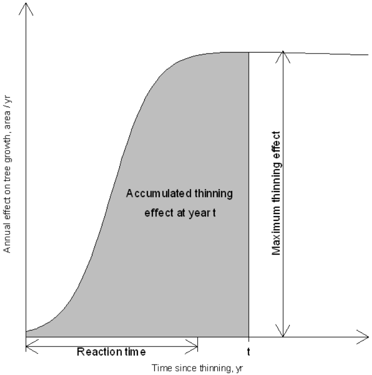
Modeling the thinning effects

- The thinning effects seem to switch on during a short time called **Reaction time** and stabilize thereafter at a level of **Maximum thinning effect**.
- To explore what predictors control these two parameters, the thinning effects of the thinnend treatments 2-4 were modeled using a nonlinear mixed-effects model.
- The random effects were used to take into account the data hierarchy for more reliable inference.

Nonlinear mixed-effects model for thinning effect

The thinning effect of tree *k* at time *t* was modeled using a logistic curve

$$d_{kt} = \frac{M_k}{1+\exp\left(4-8\frac{x_{kt}}{R_k}\right)} + e_{kt}$$



- d_{kt} - thinning effect
- x_{kt} - time since thinning
- $M_k = \mu_0 + \mu_1 T_2 + \mu_2 T_3 + \mu_4 x_{kt} + m_k$ - maximum thinning effect
- $T_2, \ldots, T_3$ - treatments
- $R_k = \rho_0 + \rho_1 z_k + r_k$ - reaction time
- z_k - standardized diameter
- $\begin{bmatrix} m_k \\ r_k \end{bmatrix} \sim MVN(\mathbf{0}, \boldsymbol{D}_{2x2})$
- e_{kt} - normal heteroscedastic residual with AR(1) structure within a tree.

The fitted model

- The reaction time was 6 years. It did not significantly vary among treatments but was shorter for large trees.

Fixed parameters	Estimate	s.e.	p-value
μ_0	112.8	23.29	0.0000
μ_1	91.91	30.45	0.0026
μ_2	169.2	32.14	0.0000
μ_3	-3.214	1.006	0.0014
ρ_0	5.749	0.4458	0.0000
ρ_1	-1.461	0.4568	0.0014
Random parameters			
var(r_k)	93.012		
var(m_k)	2.0852		
cor(r_k,m_k)	0.203		
Residual			
σ^2	8.157*10-4		
δ_1	8.746*104		
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- The maximum thinning effect increased with thinning intensity, being 282 mm/yr for treatment IV, which indicates a 87% increase in the basal area growth compared to the control.

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Motivation

- Airborne Laser Scanners (ALS) provide information on the $3D$- structure of forest
- Majority of large individual trees can be detected from an ALS point cloud
- Point cloud characteristics can be assigned to field-measured tree characteristics to estimate a system of predictive models for tree characteristics, such as stem volume, height, diameter, crown base height, dead crown height.
- These tree-specific characteristics are correlated within a forest stand
- Also the stand effects are correlated across models
- These correlations can be utilized to predict the random effects of a mixed-effects model for a given stand for all 5 models using even one observation of one characteristics only
- Enables improved predictions of hard-to-measure characteristics by using easy-to-measure characteristics

The model

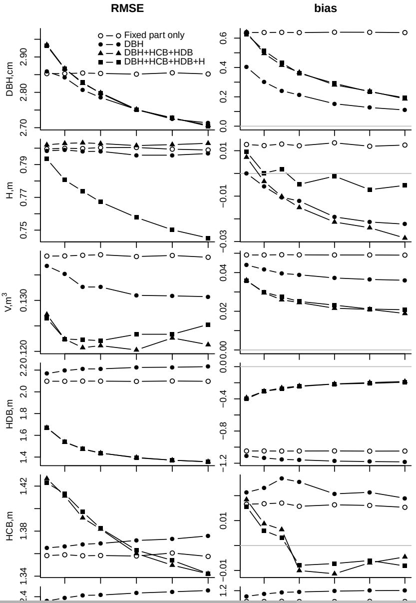
The model includes a system of 5 mixed-effects models of form for tree *i* in stand *k*:

$$
\begin{aligned}
y1_{ki} &= a1 + b1x1_{ki} + \ldots + \alpha1_k + \beta1_k x1_{ki} + \epsilon1_{ki} \\
y2_{ki} &= a2 + b1x2_{ki} + \ldots + \alpha2_k + \beta2_k x2_{ki} + \epsilon2_{ki} \\
&\vdots \\
y5_{ki} &= a5 + b5x5_{ki} + \ldots + \alpha5_k + \beta5_k x5_{ki} + \epsilon5_{ki}
\end{aligned}
$$

where the fixed parts are as with the previous mixed-effects models and include the ALS-based predictors.

- The assumptions on the random effects and residuals are $(\alpha1_k, \beta1_k, \alpha2_k, \beta2_k, \ldots, \alpha5_k, \beta5_k)' \sim MVN(0, \boldsymbol{D}_{10x10})$, and $(\epsilon1_{k1}, \epsilon2_{ki}, \ldots, \epsilon5_{ki}) \sim MVN(0, \boldsymbol{R}_{5x5})$
- The intended use of the model is prediction applying the random effects.
- The previously presented principles were used to predict the random effects of the model system by using 1-10 sample trees per stand and 3 different measurement strategies

Results


RMSE
bias
Fixed part only
DBH
DBH+HCB+HDB
DBH+HCB+HDB+H
DBH,cm
H,m
V,m3
HDB,m
HCB,m

Discussion, conclusions and references

- Mixed-effects models are useful tools for analyzing grouping datasets using models of various types.

References

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- The prediction of random effects for a **new** group is a powerful tool to localize models afterwards using very limited datasets.

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- Mixed-effects models are useful tools for analyzing grouping datasets using models of various types.
- The prediction of random effects for a **new** group is a powerful tool to localize models afterwards using very limited datasets.
- I am wondering if other fields than forestry have or could have similar applications.

References

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