Estimating forest attributes using observations of canopy height: a model-based approach

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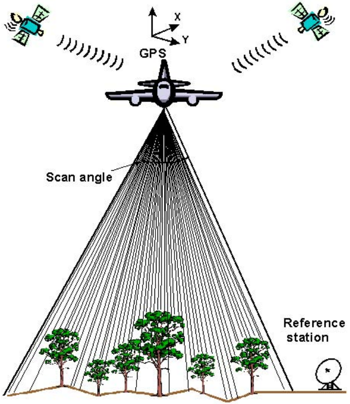
Kvantitatiivisten menetelmien seminaari
Joensuu, Finland
Based on Mehtätalo, L. and Nyblom, J. 2009. Estimating forest attributes using observations of canopy height: a model-based approach. Forest Science 55(5): 411-422.

Outline of the presentation

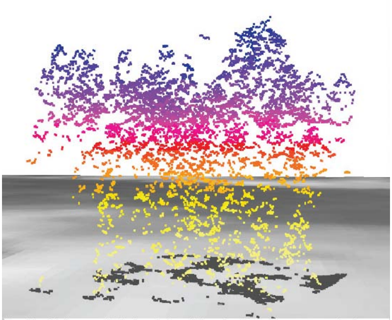
Principle of Airborne laser scanning (ALS)



from Hyyppä J, Laserkeilaimen käyttö puustotunnusten mittaamisessa.
http://www.fgi.fi/osastot/projektisivut/kk_www_portaali/rswww/lasercase1.html

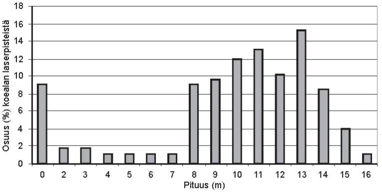
- Scan half angle 0-10 degrees.
- Footprint diameter around 0.5 meters
- Pulse density about 0.5-5 pulses per m^2.

Data obtained by ALS



From Matti Maltamo ym. 2008. Laserkeilaustulkinnan hyödyntäminen metsäsuunnittelun tietolähteenä. Metsätieteen aikakauskirja 4/2008.

Observations collected by laser scanner.



Kuva 1. Esimerkki koealan alueelle osuneista first pulse laserpisteistä muodostetusta korkeusjakaumasta.

From Suvanto et al 2005. Kuviokohtaisten puustotunnusten ennustaminen laserkeilauksella. Metsätieteen aikakauskirja 4/2005

Histogram of laser height observations from an example stand.

Current approaches

- In the **area-based approach** the forest area is divided into small grid cells, which are sampled for ground measurements. The approach is based on generalizing the estimated relationship between ground-measured and laser-scanned data from sampled cells to unsampled cells using the laser data.

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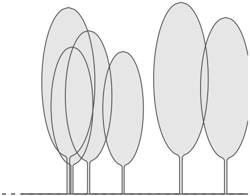
- The area-based approach can be used with low-density laser data but ground-measured sample plots are always needed. It provides fairly accurate estimates of total volume, but predicting characteristics by tree species in a mixed stand is inaccurate.
- The individual tree detection approach requires high-density laser data. However, only the largest trees can be detected, and trees forming dense groups are hard to separate. The species of recognized individual trees can be detected with sufficient accuracy when dealing with species that have different crown shapes.

Forest inventory using laser scanning



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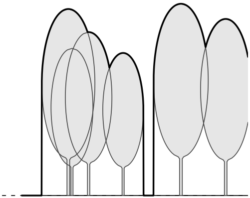

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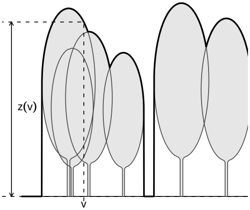

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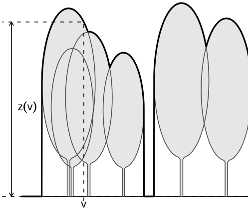

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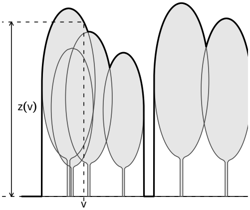

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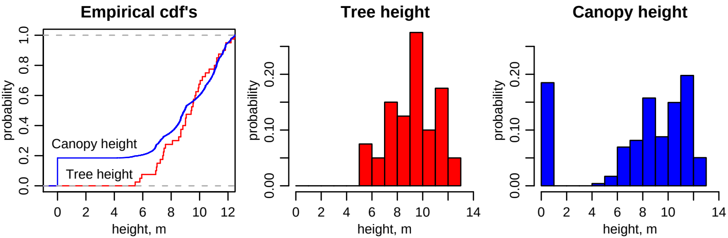
- These observations are random because their location, tree heights, and tree locations may be random.
- Several observations provide data of measured canopy heights.

Canopy height (CH) vs tree height (TH)



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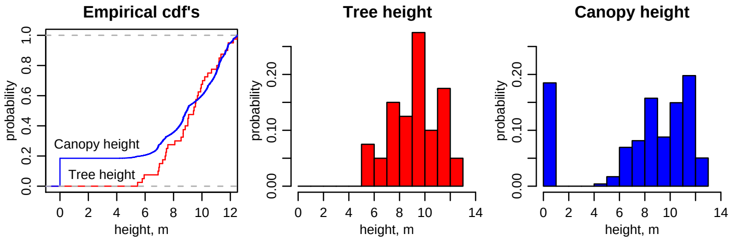

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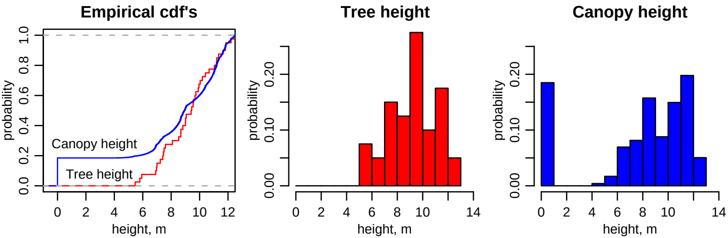

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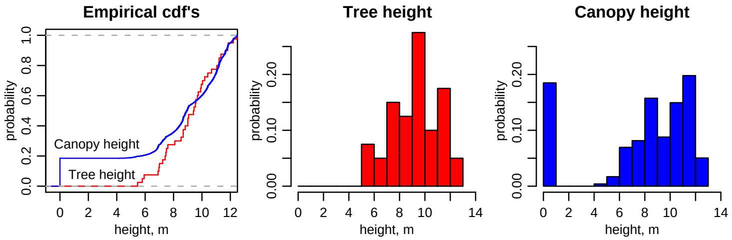

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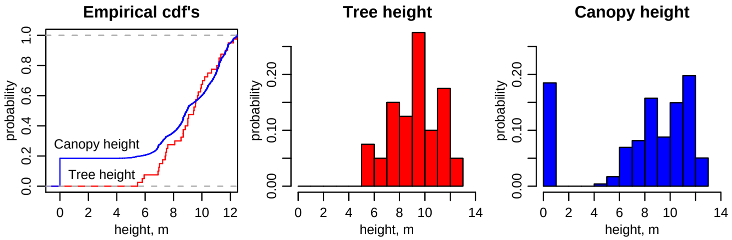

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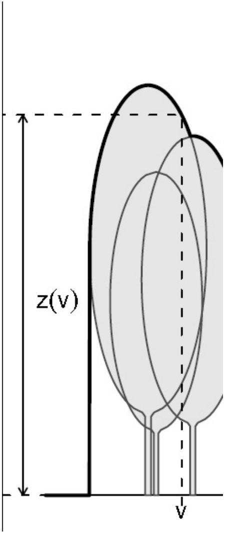
Then our question can be answered by fitting the recovered distribution to the observed data using Maximum Likelihood.

General assumptions



- Forest stand A with area $|A|$ is a realization of a stochastic model defined by
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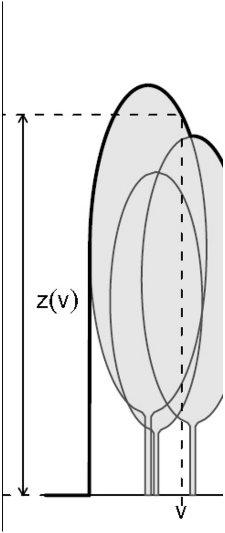

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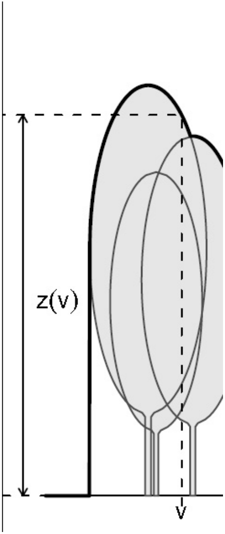

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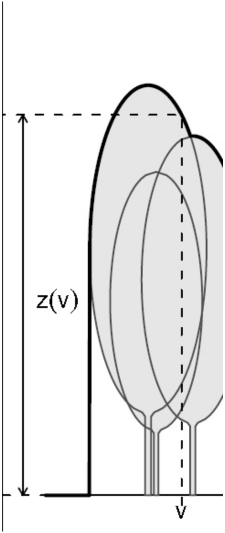

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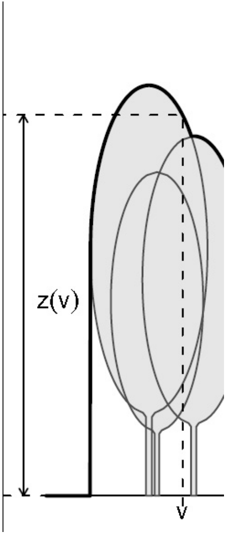

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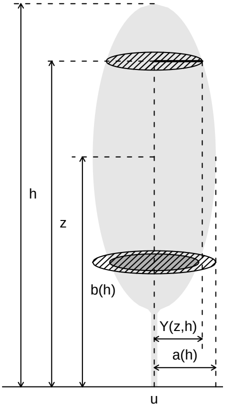
- Crown shape is a fixed, known function of tree height (e.g. ellipsoid).
- Random variable $Z(v)$ measures the vertical distance from ground level to the canopy surface, i.e., to the top of the canopy.

A model for a single tree crown



- For a tree with height h, the cross-section at height $z \geq 0$ is the set $C_0(z, h)$ (gray).
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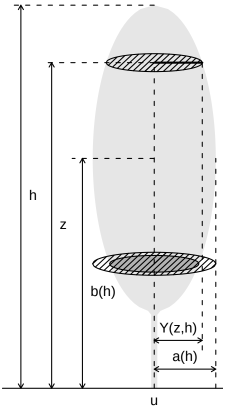

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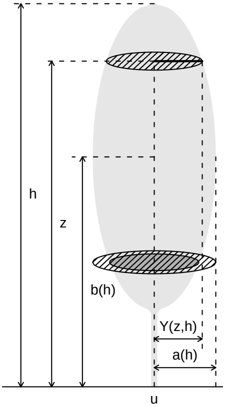

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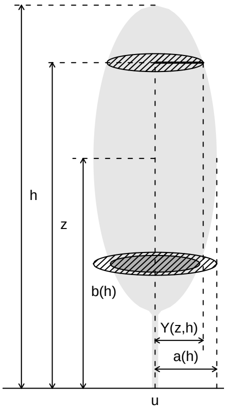

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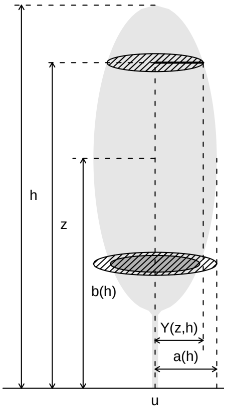

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- Denote a cross-section that is centered at u by $u + C(z, h)$.
- The asumption of symmetry guarantees that

$$P(u_2 \in u_1 + C(z, h)) \Leftrightarrow P(u_1 \in u_2 + C(z, h))$$

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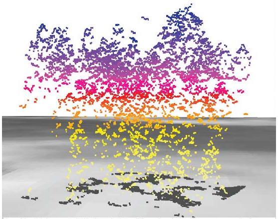
- Now $Z(v) \geq z$ if for some i $v \in u_i + C(z, H_i)$.
- In set theoretic language this means that $Z(v) \geq z \Leftrightarrow v \in \bigcup_{i=1}^{N} [u_i + C(z, H_i)]$.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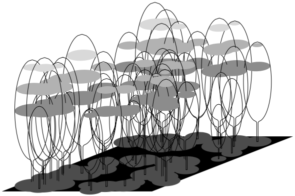

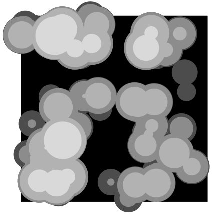

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- Because of the symmetry of cross-sections and mutual independence of tree heights, we finally get

$$
\begin{aligned}
P(Z(v) < z) &= P\left[u_i \in v + \bar{C}(z, H_i),\ \text{for all } i = 1, \ldots, N\right] \\
&= \prod_{i=1}^{N} P\left[u_i \in v + \bar{C}(z, H_i)\right],
\end{aligned}
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  - Finally, the expectation over N yields the result.

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- We start with $E\left[E\left\{P\left(\bigcap_{i=1}^{N}[u_i \in v + \bar{C}(z, H_i)]\right) \,\middle|\, N\right\}\right].$

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- By continuity of $|C(z, h|$ we have $P(Z(v) < z) = P(Z(v) \leq z)$, when $z > 0$.

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- By continuity of $|C(z,h|$ we have
  $P(Z(v)<z) = P(Z(v)\leq z)$, when $z>0$.
- At $z=0$ we have a point mass $P(Z(v)=0) = \exp\{-\lambda E(|C(0,H)|)\}$, but naturally $P(Z(v)<0)=0$.

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- At $z=0$ we have a point mass $P(Z(v)=0)=\exp\{-\lambda E(|C(0,H)|)\}$,
  but naturally $P(Z(v)<0)=0$.
- By homogeneity of the Poisson stand, $P(Z(v)\leq z)$ applies to all locations $v \in A$.

Distribution function and density

- The distribution function is
$P(Z \leq z) = G(z|\lambda, \xi) = \exp\left\{-\lambda \int_0^\infty |C(z,h)| f(h|\xi) \mathrm{d}h\right\}$

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- The distribution function is the porosity of the model as a function of reference height z.

Estimation

- The parameters are estimated by using the method of maximum likelihood. The log likelihood under independnce of observations is

$$\ell(\boldsymbol{\xi}, \lambda) = \sum_{j=1}^{M} I(z_j > 0) \log g(z_j|\boldsymbol{\xi}, \lambda) + M_0 \log G(0|\boldsymbol{\xi}, \lambda) \,,$$

where I is the indicator function and M_0 is the number of ground hits (for which $z_j = 0$).
- The values of λ and $\boldsymbol{\xi}$ that maximize the log likelihood are the ML-estimates for stand density, and parameters of the height distribution. Asymptotic properties of ML-estimator can be used for to assess their accuracy and make inference.

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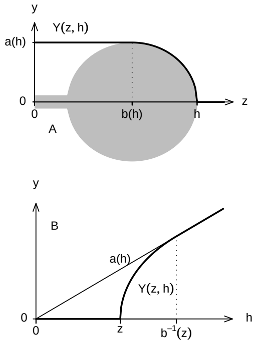
- We assume independence of observations, which holds only for low spacing (no several observations per tree).
- With dense spacing, estimates will still be asymptotically unbiased, but the standard errors will be downward biased. However, more efficient estimators maybe available that utilize the spatial autocorrelation of observations.

Simulated data



- We assumed that $H \sim Weibull(\alpha, \beta)$.
- Tree crowns were assumed ellipsoids with circular cross-section. Then $|C(z,h)| = \pi Y(z,h)^2$ and the squared crown radius is

$$Y(z,h)^2 = \begin{cases} 0, & 0 \leq h < z, \\ a(h)^2 \left(1 - \frac{\{z-b(h)\}^2}{\{h-b(h)\}^2}\right), & z \leq h < b^{-1}(z), \\ a(h)^2, & h \geq b^{-1}(z). \end{cases}$$

where $a(h) = ph$ and $b(h) = qh$ with $p = 0.1$ and $q = 0.6$.

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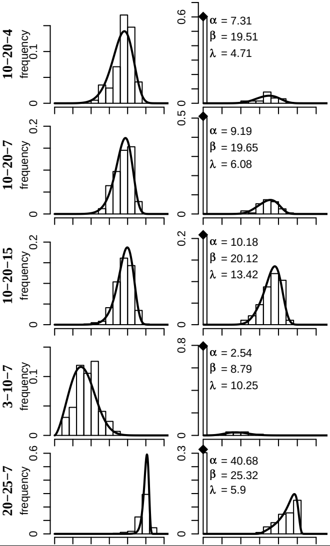
- The model was fitted using R-package function `mle in package stats4`.

Example fits



One randomly selected fit for each of the five parameter combinations with the sample size of 100. The right panel shows the observations of canopy height (histogram) and the fitted density (solid line and diamond). The left panel shows the true tree heights (histogram) and the estimated distribution of the same plot. The numbers show the parameter estimates.

Simulation results

Bias of estimates of mean height (meters) and stand density (trees per ha) in different simulations.

		M=30		M=100		M=400	
α-β-λ		$\bar{H}$	N	$\bar{H}$	N	$\bar{H}$	N
10-20-4	true mean	19.03	400	19.03	400	19.03	400
	bias%	0.84	2.26	0.62	0.09	0.12	0.30
	s.d.	1.21	148.53	0.67	78.16	0.36	53.56
10-20-7	true mean	19.03	700	19.03	700	19.03	700
	bias%	0.89	2.42	0.45	-1.66	0.07	0.43
	s.d.	1.03	235.62	0.56	125.15	0.31	79.85
10-20-15	true mean	19.03	1500	19.03	1500	19.03	1500
	bias%	0.02	13.27	-0.05	2.76	-0.07	0.70
	s.d.	1.08	2698.26	0.53	263.39	0.29	145.76
3-10-7	true mean	8.93	700	8.93	700	8.93	700
	bias%	6.65	1.76	1.45	3.20	0.65	-0.57
	s.d.	2.27	480.08	1.24	238.60	0.66	122.87
20-25-7	true mean	24.34	700	24.34	700	24.34	700
	bias%	0.23	1.28	-0.08	2.12	0.02	0.40
	s.d.	0.78	205.81	0.43	112.24	0.22	73.31

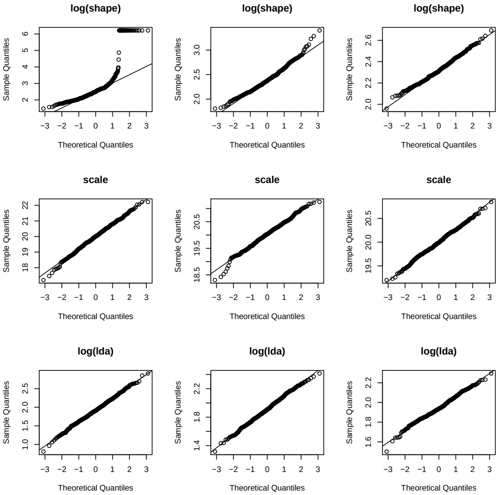
M=number of observations, α and β Weibull parms, λ true number of stems. Sample plot area was 2000m^2

Bias, standard deviation and asymptotic s.e. of parameter estimates.

		M=30			M=100			M=400		
		$\log(\widehat{\alpha})$	$\widehat{\beta}$	$\log(\widehat{\lambda})$	$\log(\widehat{\alpha})$	$\widehat{\beta}$	$\log(\widehat{\lambda})$	$\log(\widehat{\alpha})$	$\widehat{\beta}$	$\log(\widehat{\lambda})$
10-20-4	true mean	2.30	20	1.39	2.30	20	1.39	2.30	20	1.39
cc=0.37	bias	>0.967	-0.118	-0.046	>0.139	0.042	-0.019	0.034	-0.002	-0.006
	s.d.	1.580	0.988	0.379	0.422	0.545	0.200	0.136	0.310	0.134
	$\bar{\hat{\sigma}}$	1.179	0.824	0.348	0.296	0.514	0.187	0.126	0.263	0.093
10-20-7	true mean	2.30	20	1.95	2.30	20	1.95	2.30	20	1.95
cc=0.55	bias	>0.509	-0.011	-0.027	0.093	0.025	-0.033	0.018	0.002	-0.002
	sd	1.127	0.814	0.320	0.241	0.459	0.181	0.109	0.266	0.113
	$\bar{\hat{\sigma}}$	0.718	0.748	0.295	0.221	0.435	0.161	0.103	0.222	0.080
10-20-15	true mean	2.30	20	2.71	2.30	20	2.71	2.30	20	2.71
cc=0.82	bias	>0.162	-0.078	0.014	0.033	-0.031	0.013	0.005	-0.016	0.002
	sd	0.535	0.894	0.342	0.192	0.423	0.165	0.097	0.242	0.096
	$\bar{\hat{\sigma}}$	0.398	0.759	0.294	0.180	0.408	0.154	0.087	0.204	0.076
3-10-7	true mean	1.10	10	1.95	1.10	10	1.95	1.10	10	1.95
cc=0.18	bias	>1.221	0.226	-0.202	0.099	0.056	-0.024	0.033	0.038	-0.021
	sd	2.005	2.252	0.693	0.333	1.254	0.341	0.157	0.685	0.179
	$\bar{\hat{\sigma}}$	1.460	1.700	0.630	0.288	1.188	0.336	0.139	0.609	0.167
20-25-7	true mean	3.00	25	1.95	3.00	25	1.95	3.00	25	1.95
cc=0.73	bias	>0.640	-0.082	-0.027	0.054	-0.037	0.009	0.017	-0.001	-0.001
	sd	1.199	0.569	0.279	0.270	0.321	0.155	0.129	0.172	0.104
	$\bar{\hat{\sigma}}$	0.953	0.520	0.259	0.266	0.313	0.141	0.124	0.156	0.070

M=number of observations, α and β Weibull parms, λ true number of stems. Sample plot area was 2000m^2

Normal-Q-Q-plots of estimates

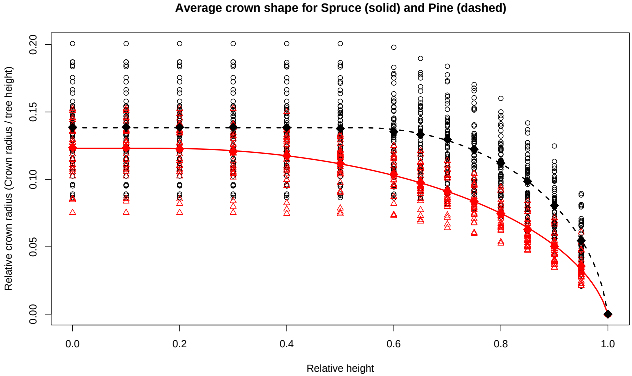


Normal Q-Q plots of estimates for the stand 10-20-7 with sample sizes of 30 (left) 100 (middle) and 400 (right).

Tests with real data

- Laser data 40 pulses/m^2 and aerial photographs from Tielaitos.
- Ground data (20 plots of size 20*20m). Species and dbh known for each tree.
- Single tree from 18 plots were used for modeling crown shape (the models I showed earlier)
- Thinned "laser" data (0.25 pulsesper m^2) of the rest two plots were used for method testing.

Models for crown shape



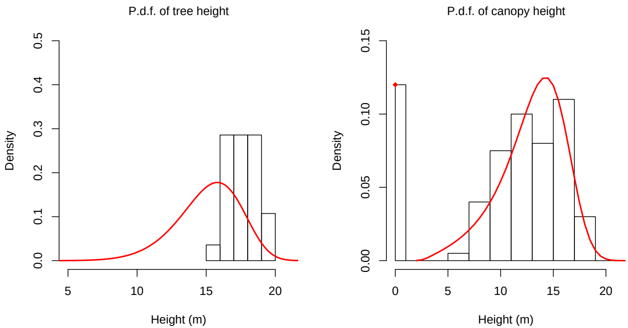
$$\frac{r}{H} = \begin{cases} y_0 + b & \frac{h}{H} \leq x_0 \\ y_0 + b\sqrt{1 - \frac{(\frac{h}{H} - x_0)^2}{a^2}} & x_0 < \frac{h}{H} \leq 1 \\ 0 & \frac{h}{H} > 1 \end{cases} \quad \text{where } a = \sqrt{\frac{b^2(1-x_0)^2}{b^2 - y_0^2}}$$

Evaluation plots

Pure spruce stand
$\bar{H}$=17.77 m
N=700 stems/ha

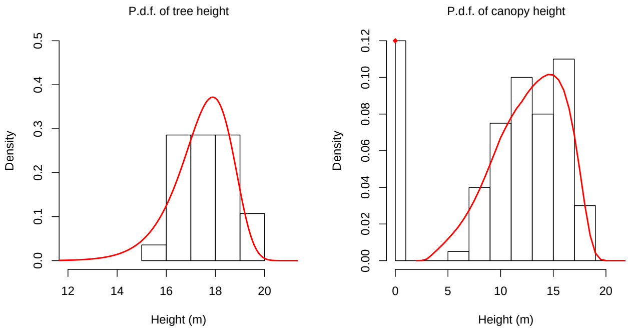
Mixed spruce-pine stand
$\bar{H}_{spruce}$=9.87 m
$\bar{H}_{pine}$=15.66 m
N=1350 stems/ha
ρ=0.70 (70% were spruces)

Plot 8 by assuming random locations



$\widehat{\lambda}$=1943 r/ha (true 700)
$\widehat{\bar{H}}$=15.12 m (true 17.77)

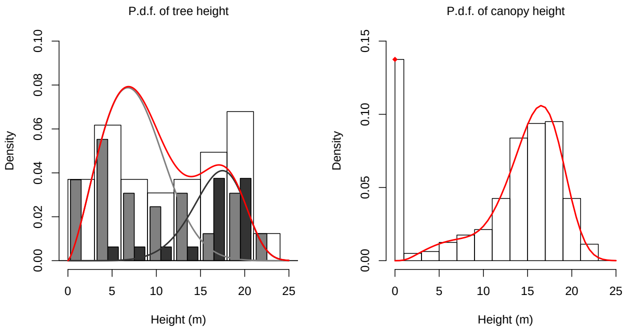
Plot 8 by assuming square grid locations



$\widehat{\lambda}$=642 r/ha (true 700)

$\widehat{\bar{H}}$=17.52 m (true 17.77)

Plot 28 by assuming random locations



$\widehat{\lambda}$=2645 r/ha (true 1350)

$\widehat{\rho}$=0.6998 r/ha (true 0.7037)

$\widehat{\bar{H}_{spruce}}$=7.57 (true 9.87)

$\widehat{\bar{H}_{pine}}$=16.72 (true 15.66)

Discussion and conclusions

- First trials to derive stand characteristics by utilizing assumptions on crown shape, and a spatial model for the stand.
- ML-estimation for the model seems to be quite stable and produces quite reliable estimates with simulated datasets
- Comutationally intensive method as it needs nested application of numerical methods
- Seems to be quite vulnerable to violation of asumptions on spatial pattern of tree locations
- Could be a way to integrate the research on single tree crowns with the models of stand structure.
- Could provide a theoretical basis for the so-callet area-based approach.

Publications

Mehtätalo, L. and Nyblom, J. Estimating forest parameters using observations of canopy height: a model-based approach. Forest Science 55(5): 411-422.

Mehtätalo, L. 2006. Eliminating the effect of overlapping crowns from aerial inventory estimates. Canadian Journal of Forest Research 36(7): 1649-1660.
(Reprints available upon request)