Predicting stand structure using limited measurements

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Outline of the presentation

Introduction

Methods
- Linear Prediction
- Examples of BLP

Predicting the distribution of diameters
- Percentile-based diameter distribution
- Distributions of order statistics
- BLP in this case
- The effect of order statistics on prediction accuracy

Discussion

The inventory of a single stand

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- The main aim of this study was to develop tools for producing a stand description from ground-measured data
- Because of limited measurement resources for a single stand, the available information is very limited and the use of measurement information should be as effective as possible.

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Group2: Sample measurements (of response)

- Sample tree height(s)
- Sample order statistic(s) of the angle count sample plot(s), called quantile trees.

The use is based on **linear prediction**.

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The Best Linear Predictor (BLP) of $\mathbf{x}_1$ is

$$\widehat{\mathbf{x}_1} = \boldsymbol{\mu}_1 + \mathbf{V}_{12}\mathbf{V}_2^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2)$$

and the variance of prediction errors is

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In applications, we usually use estimated $\boldsymbol{\mu}$'s and $\mathbf{V}$'s to obtain an Estimated BLP (EBLP).

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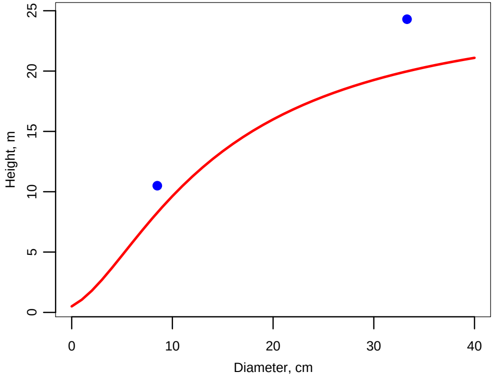
- In spatial statistics, kriging is a method where unobserved responses, $\mathbf{x}_1$ at spatial locatiobns are predicted using observed responses, $\mathbf{x}_2$ at spatial locations. Matrices $\mathbf{V}$ are obtained from an estimated variogram.

Predicting Height-Diameter curve

The fixed part of a mixed model gives the expected height for a tree with a given diameter. If we have sample tree heights measured, we can predict the random effects of the model ($\mathbf{x}_1$) using sample tree heights ($\mathbf{x}_2$)



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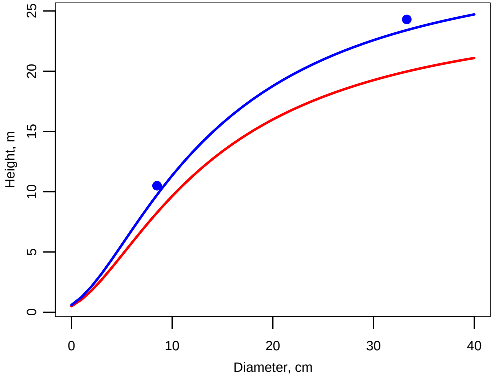

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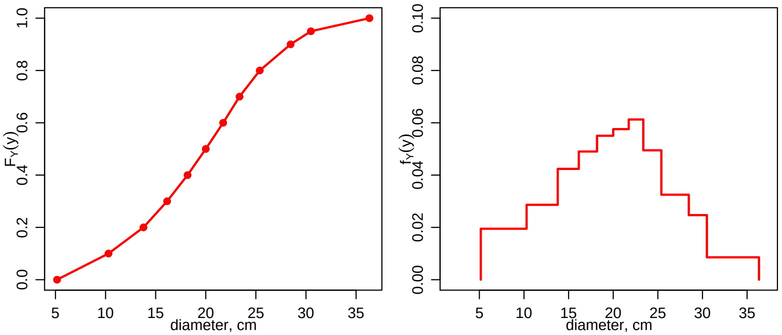

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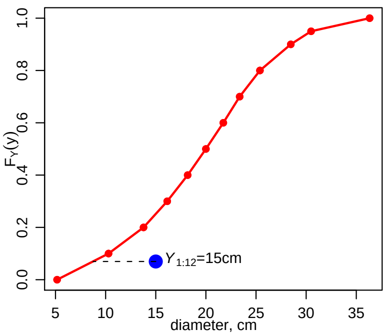

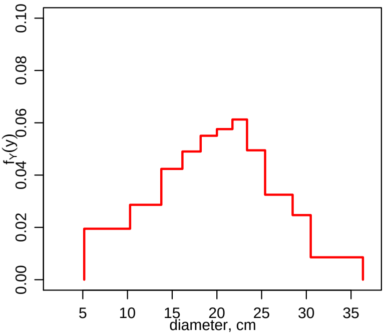

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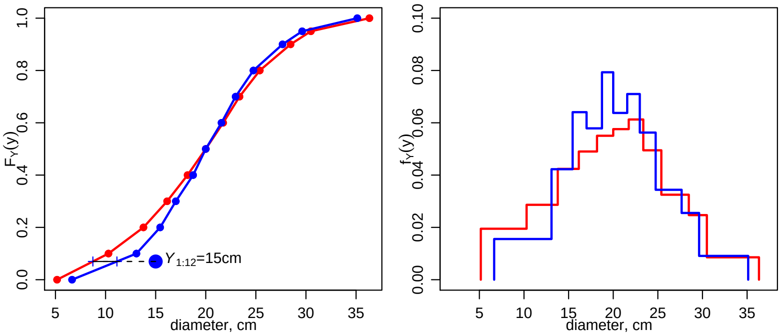
- so we can use it ($\mathbf{x}_2$) to predict the residuals of our percentile models ($\mathbf{x}_1$).



Percentile-based diameter distribution

Certain percentiles in stand k are modeled using Group 1 variables

$$
\begin{aligned}
d_{0\%,k} &= E(d0\%|DGM_k, G_k, T_k, \ldots) + e_{0\%,k} \\
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The same in matrix form:

$$\mathbf{d}_k = E(\mathbf{d}|DGM_k, G_k, T_k, \ldots) + \mathbf{e}_k \,,$$

where residuals $\mathbf{e}_k$ are actually stand effects with nondiagonal $\mathrm{var}(\mathbf{e}_k) = \mathbf{D}$. We will drop index k hereafter.

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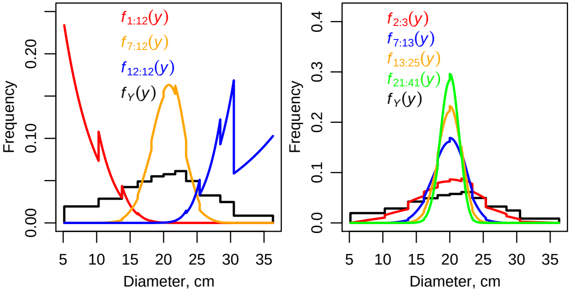
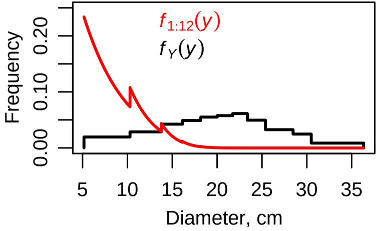


Examples of distributions, black line is the underlying population distribution

Expectation and variance of $Y_{r:n}$

Recall the example, where $Y_{1:12}$ was observed

Assuming that the predicted $f_Y(y)$ is the diameter distribution, we know the distribution of $Y_{1:12}$.



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$E(Y_{1:12}) = 8.73\text{cm}$ and
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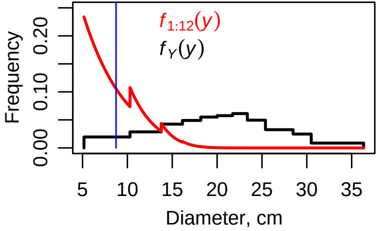

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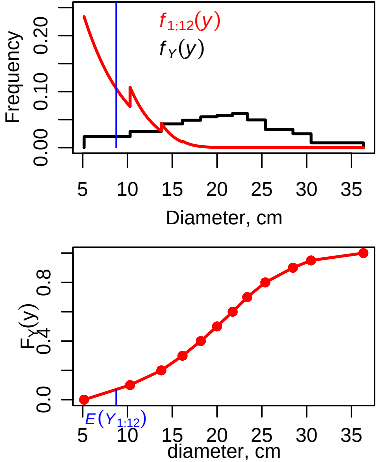

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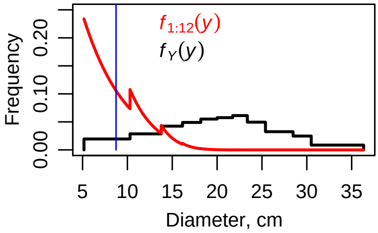

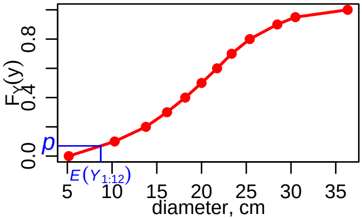

The model for the measured percentile

Recall the model for the predefined percentiles

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where $\mathrm{var}(\mathbf{e}_k) = \mathbf{D}$.
The model for this observation is

$$d_{7\%} + \epsilon = E(d_{7\%}|DGM, G, T, \ldots) + e_{7\%} + \epsilon \,,$$

where ϵ is the measurement error of of the percentile, with $\mathrm{var}(\epsilon) = \mathrm{var}(Y_{1:12}) = 8.09$.
We approximate $E(d_{7\%}|DGM, G, T, \ldots)$, $\mathrm{var}(e_{7\%})$ and $\mathrm{cov}((e), e_{7\%})$ by interpolating linearly $E(\mathbf{d}|DGM, G, T, \ldots)$ and $\mathbf{D}$ for the 7th percentile.

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We can now write

$$\begin{bmatrix} \mathbf{d} \\ d_{7\%} + \epsilon \end{bmatrix} \sim \left(\begin{bmatrix} E(\mathbf{d}|DGM, G, T, \ldots) \\ E(d_{7\%}|DGM, G, T, \ldots) \end{bmatrix}, \begin{bmatrix} \mathbf{D} & \mathrm{cov}(\mathbf{e}, e_{7\%}) \\ \mathrm{cov}(e_{7\%}, \mathbf{e}) & \mathrm{var}(e_{7\%}) + \mathrm{var}(\epsilon) \end{bmatrix} \right)\,,$$

where all except for $\mathbf{d}$ is known.

BLP of stand effects

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The BLP is calculated as

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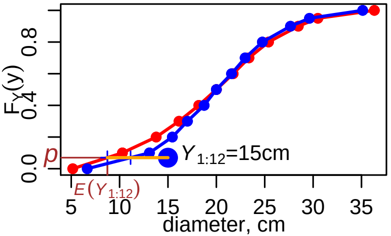

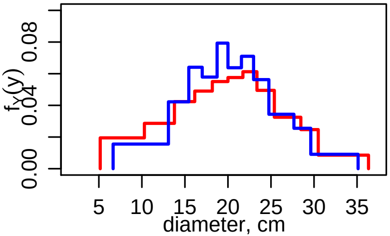

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- Our $f_{r:n}(y)$ was based on assumption that $E(\mathbf{d}|DGM, G, T, \ldots)$ gives the *true* distribution of the stand.
- Now we have a better estimate for the true distribution, namely $\widehat{\mathbf{d}}$.
- That is why we use $\widehat{\mathbf{d}}$ to calculate p and $\mathrm{var}(Y_{r:n})$ again, and predict new $\widehat{\mathbf{d}}$.

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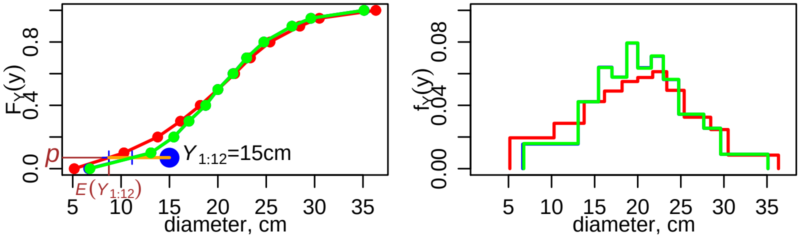
After 7 iterations the final values were $p = 0.064$, $E(Y_{1:12}) = 10.82\mathrm{cm}$ and $\mathrm{var}(Y_{r:n}) = 8.28$.



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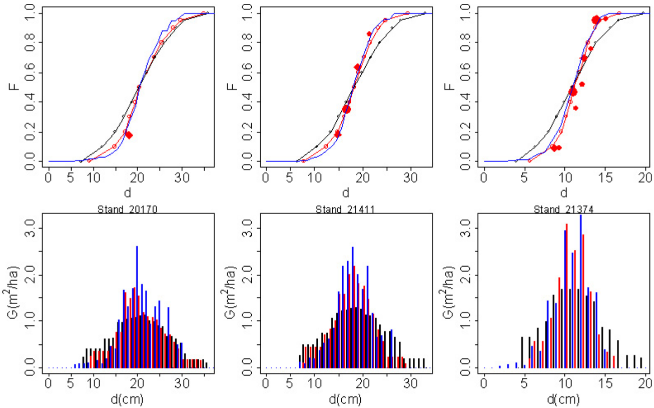
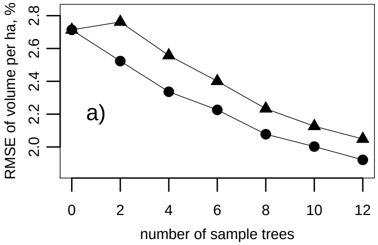
- In the case of several observation per plot, ϵ's are correlated. Joint distribution of two order statistics is needed to calculate the correlation:

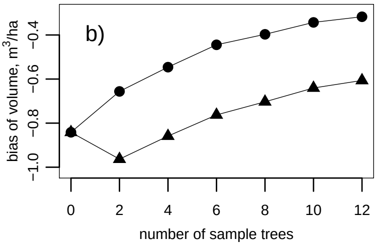
Examples of predictions (True trees from true plots)



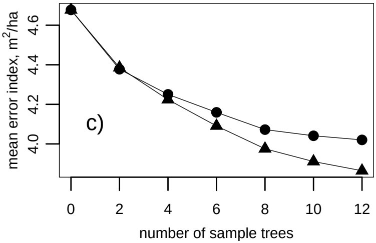
True distribution, prediction based on *DGM*, *G*, *T*, *soil* and the prediction based on *DGM*, *G*, *T*, *soil* and sample order statistics (the marks). Same mark is used for measurements from same plot.

True trees from simulated plots



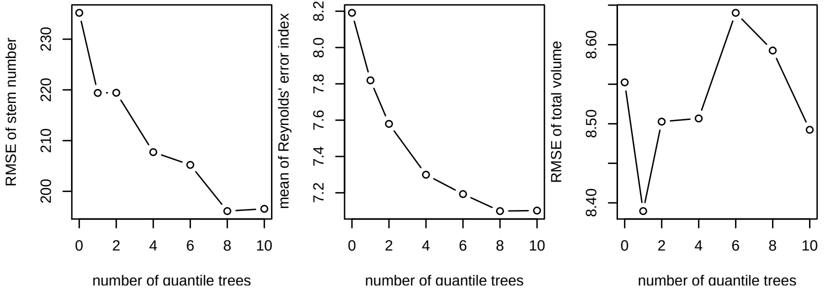


- Order statistics are taken from *random samples* of Norway spruce stands.
- Triangles show the prediction after first step, the circle after converged iteration.

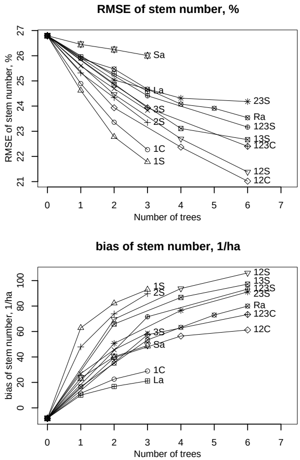


True trees from true plots

- Trees were randomly selected from 3 angle count sample plots
- Stand variables (DGM, G, N) included measurement errors
- Observations in a sample seem not to be independent observations from the population



Which quantile to measure?



Quantile trees were selected according to different strategies from 1-3 angle count plots

- 1S, 2S, 3S: 1st, 2nd and 3rd smallest tree of each plot
- 12S, 13S, 23S: 1st and 2nd, 1st and 3rd, and 2nd and 3rd smallest trees of each plot
- 1C, 2C, 3C: 1st, 2nd and 3rd closest tree of each plot
- 12C, 13C, 23C: 1st and 2nd, 1st and 3rd, and 2nd and 3rd closest trees of each plot
- Sa: Tree closest to sawtimber limit (17 cm in diameter)
- La: Largest tree of each plots
- Ra: Randomly selected trees

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- We used BA diameter distribution and angle count samples. With unweighted distributions we would need to use fixed area samples.
- Here we worked with percentile-based distribution. What about other families, such as Weibull

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- Here we worked with percentile-based distribution. What about other families, such as Weibull
  - Modeling simultaneously Weibull-parameters and percentiles

Discussion

- Using sample information combined with models provides possibilities to control the accuracy of a forest inventory according to the information needs
- Which quantiles are the easiest to measure?
- Using extremes seems to cause a bias problem
- We used BA diameter distribution and angle count samples. With unweighted distributions we would need to use fixed area samples.
- Here we worked with percentile-based distribution. What about other families, such as Weibull
    - Modeling simultaneously Weibull-parameters and percentiles
    - Combining PPM-weibull and a Weibull fitted to a small sample by ML

Publications

Mehtätalo, L. 2005. Localizing a predicted diameter distribution using sample information. Forest Science 51(4): 292-302.

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