Predicting the structure of growing stock using limited measurements

Overview of my dissertation

Lauri Mehtätalo
lauri.mehtatalo@yale.edu

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Outline of the presentation

The inventory of a single stand

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- The main aim of this study was to develop tools for producing a stand description from ground-measured data
- Because of limited measurement resources for a single stand, the available information is very limited and the use of measurement information should be as effective as possible.

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- Some basic information (stand age, site fertility class ...)
- Basal area and basal area weighted median diameter (DGM) from angle count plot(s)

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- Sample tree height(s)
- Sample order statistic(s) of the angle count sample plot(s), called quantile trees.

The use is based on **linear prediction.**

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In applications, we usually use estimated $\boldsymbol{\mu}$'s and $\mathbf{V}$'s to obtain an Estimated BLP (EBLP).

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- In co-kriging, $\mathbf{x}_2$ is augmented with the values of the correlated auxiliary variable(s) at the points being predicted.

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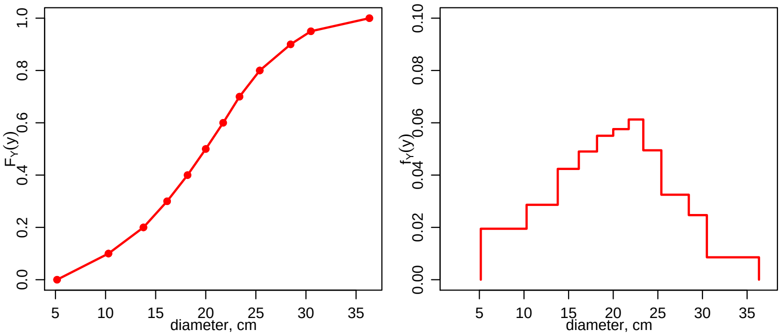
- I predict diameter percentiles using models that are estimated from some data *a priori*.



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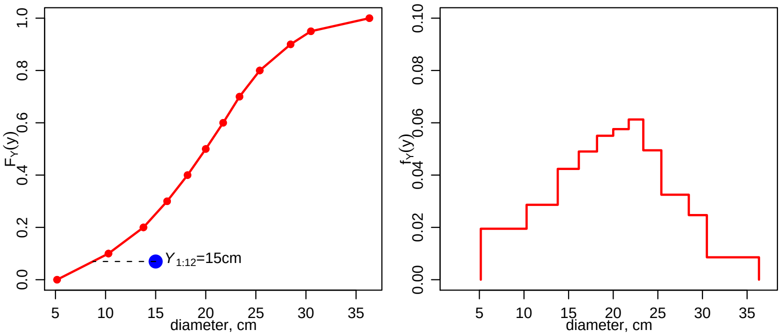

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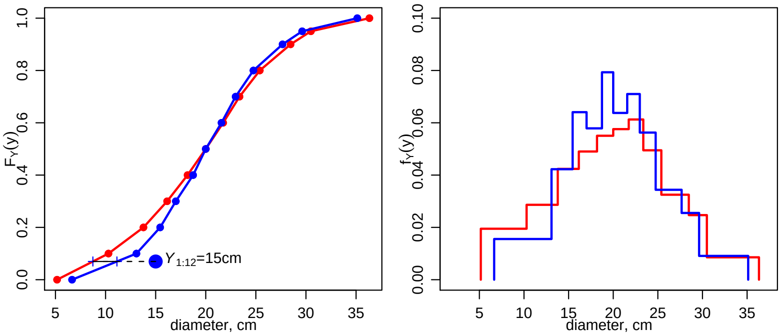

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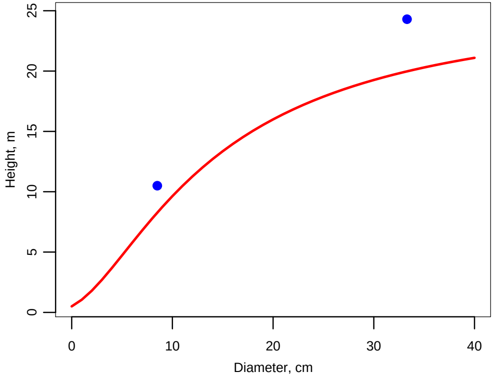

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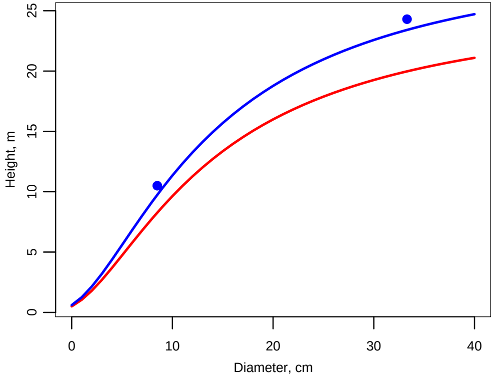

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- estimate all cross-model correlations, so as to make it possible to use all correlations between stand effects.
- add a model for the development of basal area, so as to complete the system as a growth and yield model that can effectively utilize measurements from various points in time and also can make predictions into the future