DYNAMIC BOOLEAN MATRIX FACTORIZATIONS

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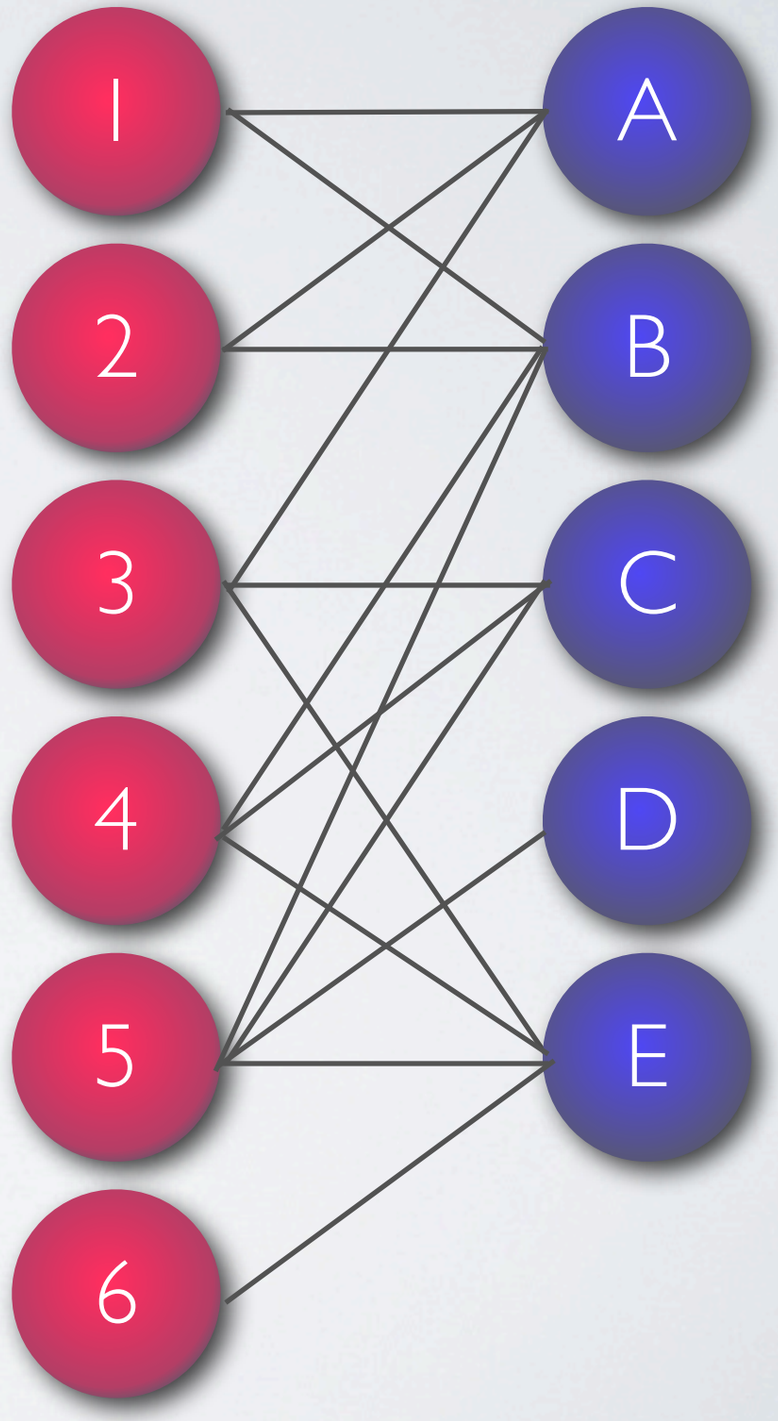
USERS AND WEBPAGES

- Users "like" webpages
  - A bipartite graph
- We want to know (approximate) bicliques of users who like similar webpages



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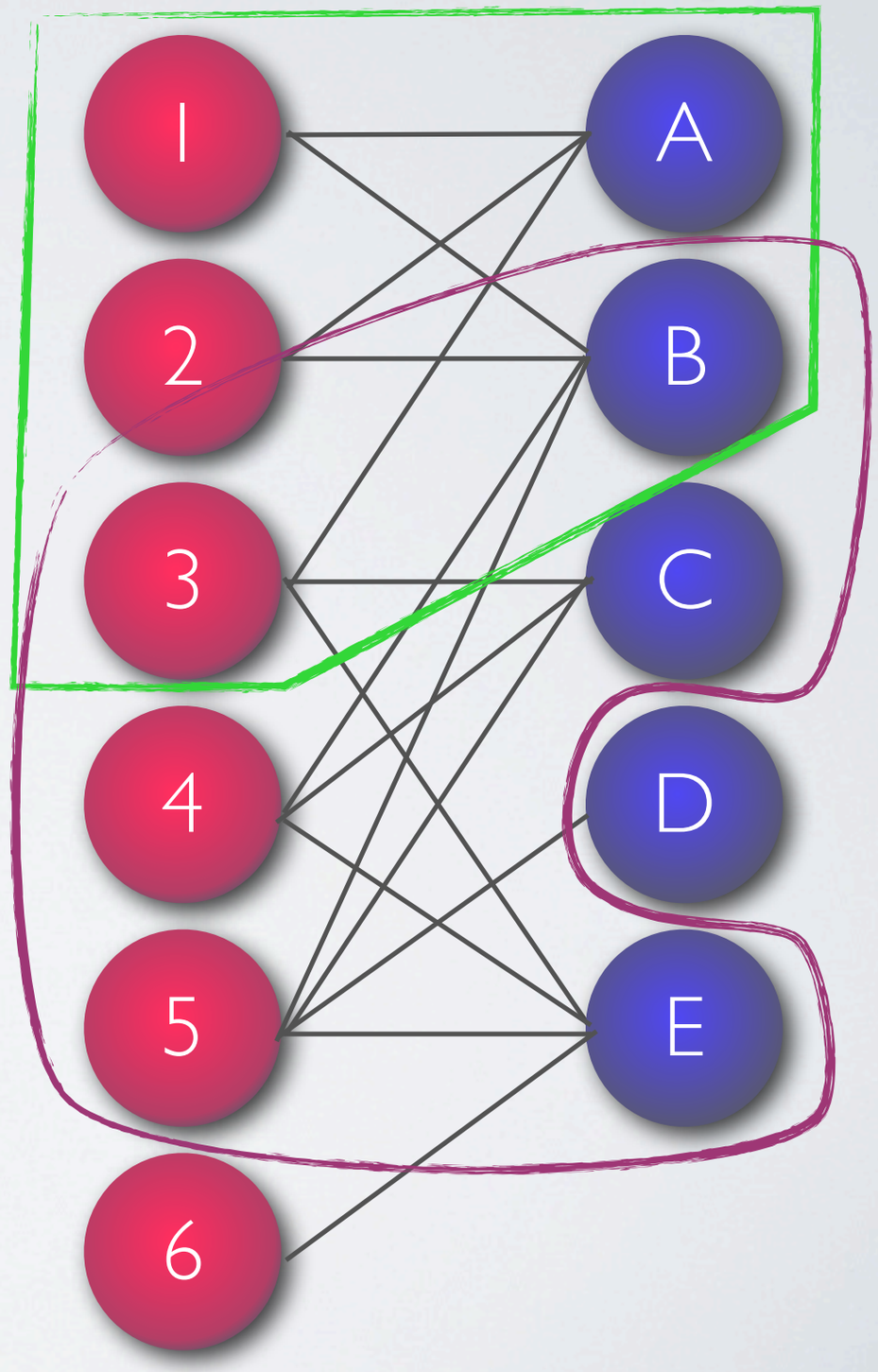
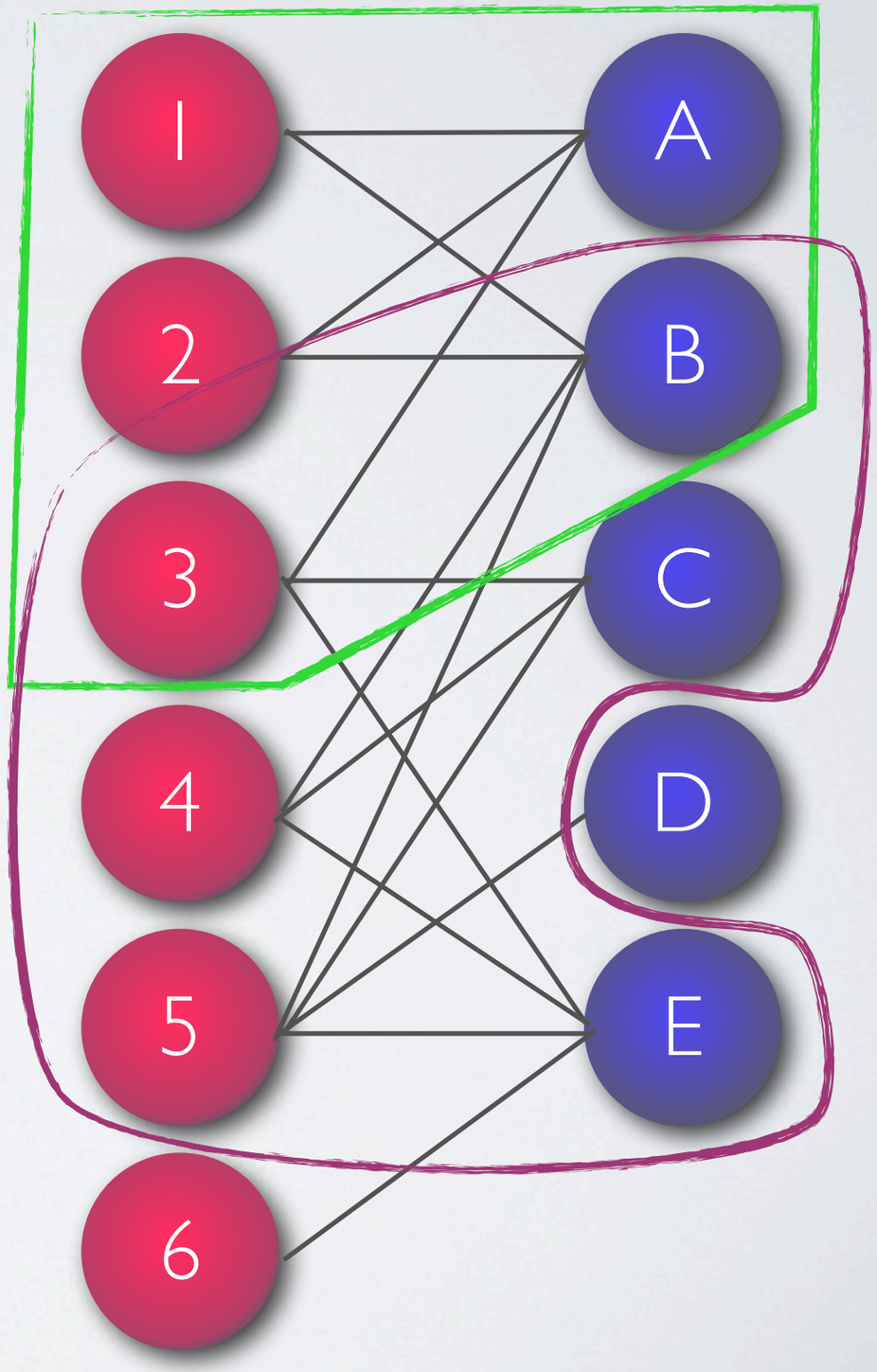
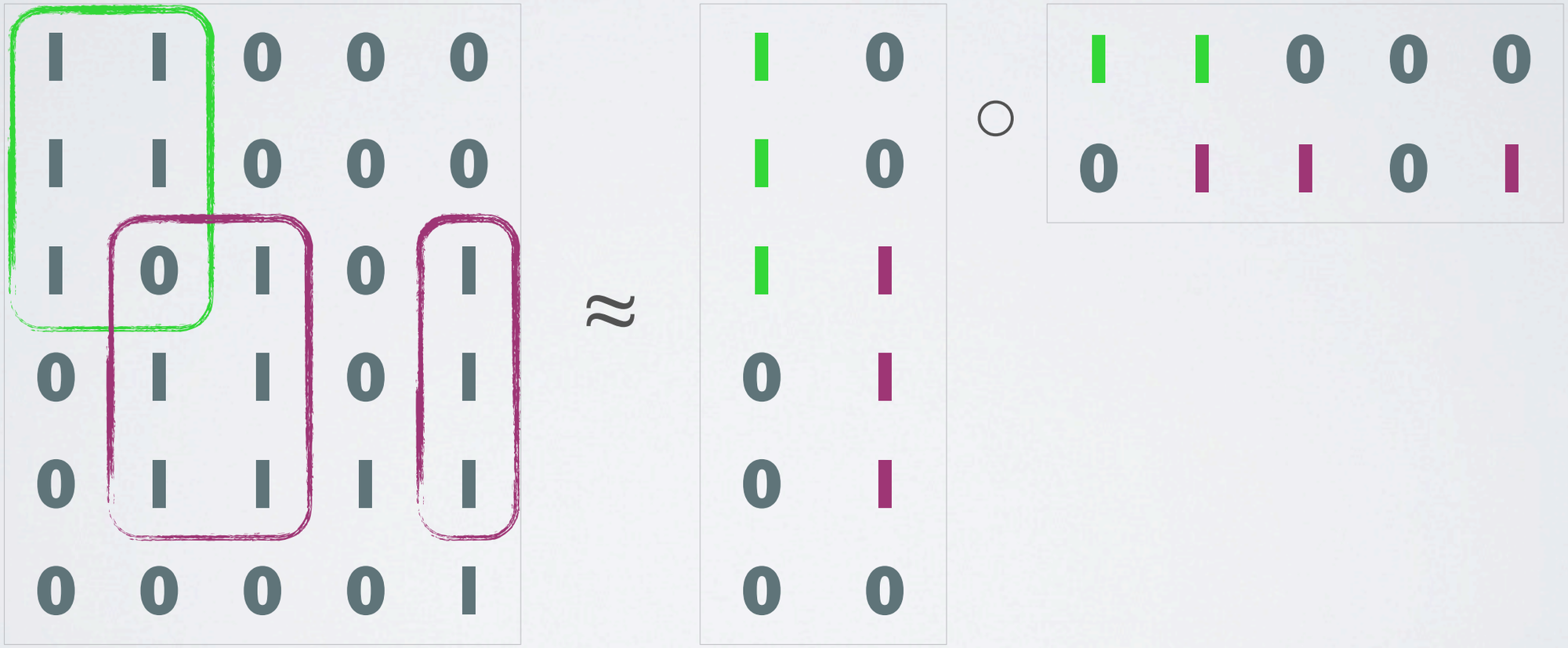


BOOLEAN MATRIX

1	1	0	0	0
1	1	0	0	0
1	0	1	0	1
0	1	1	0	1
0	1	1	1	1
0	0	0	0	1



BOOLEAN MATRIX FACTORIZATIONS



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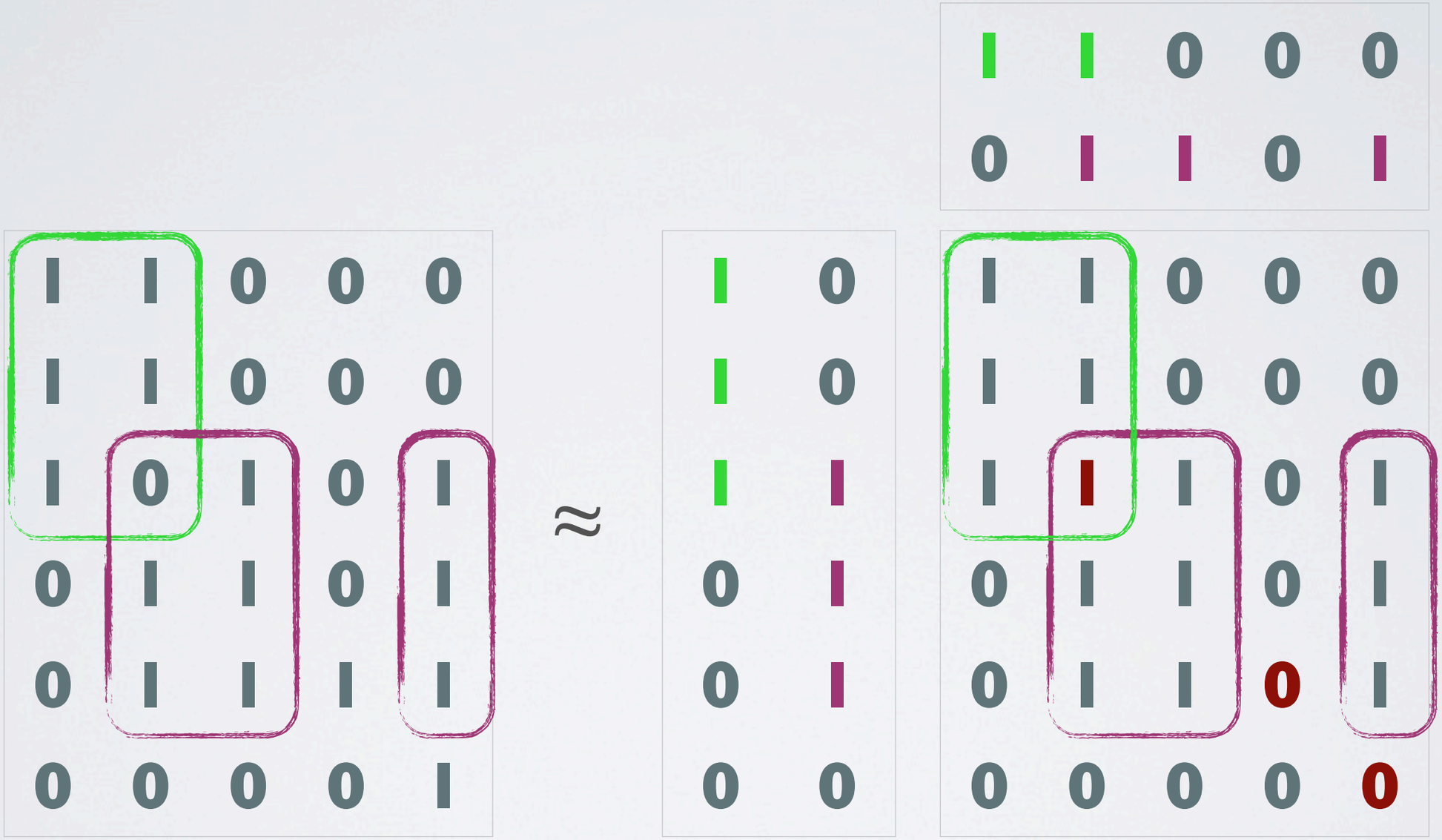

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DYNAMIC FACTORIZATIONS

- Users keep on liking new webpages
  - New users, new webpages, and old users liking old pages
- We want our factorization to adapt to new data
- **Problem.** Given a binary matrix ***A***, its Boolean factorization (***B***, ***C***), and a series of added 1s to ***A***, update ***B*** and ***C*** so that they define a good approximation of ***A*** after any addition

NOTES

- We're only adding 1s to the data
  - You can't dislike a page
- We're **not** doing prediction, we're adapting
  - Being good at predicting helps adapting, though

FIRST ATTEMPT

- We can re-compute the factorization after every addition
- Too slow
- Too much effort given the minimal change

SECOND ATTEMPT

- We can fold-in the new data: if we add a new column to ***A***, we keep ***B*** fixed and add a new column to ***C***
  - Common in IR when new terms/documents appear
- But we're not necessarily adding new rows or columns
  - We could still do alternating updates to ***B*** and ***C***, except that the problem is NP-hard to even approximate well

THIRD ATTEMPT

- An **online** algorithm
  - Will never remove any 1s it has added to factor matrices
- We consider three cases when a new 1 arrives

EXAMPLE

1	1	0	0	0
0	1	1	0	1

1	1	0	0	0
1	0	1	0	1
0	1	1	0	1
0	1	1	1	1
0	0	0	0	1

≈

1	0
1	1
0	1
0	1
0	0

1	1	0	0	0
1	1	1	0	1
0	1	1	0	1
0	1	1	0	1
0	0	0	0	0

CASE 1: PREDICTED 1

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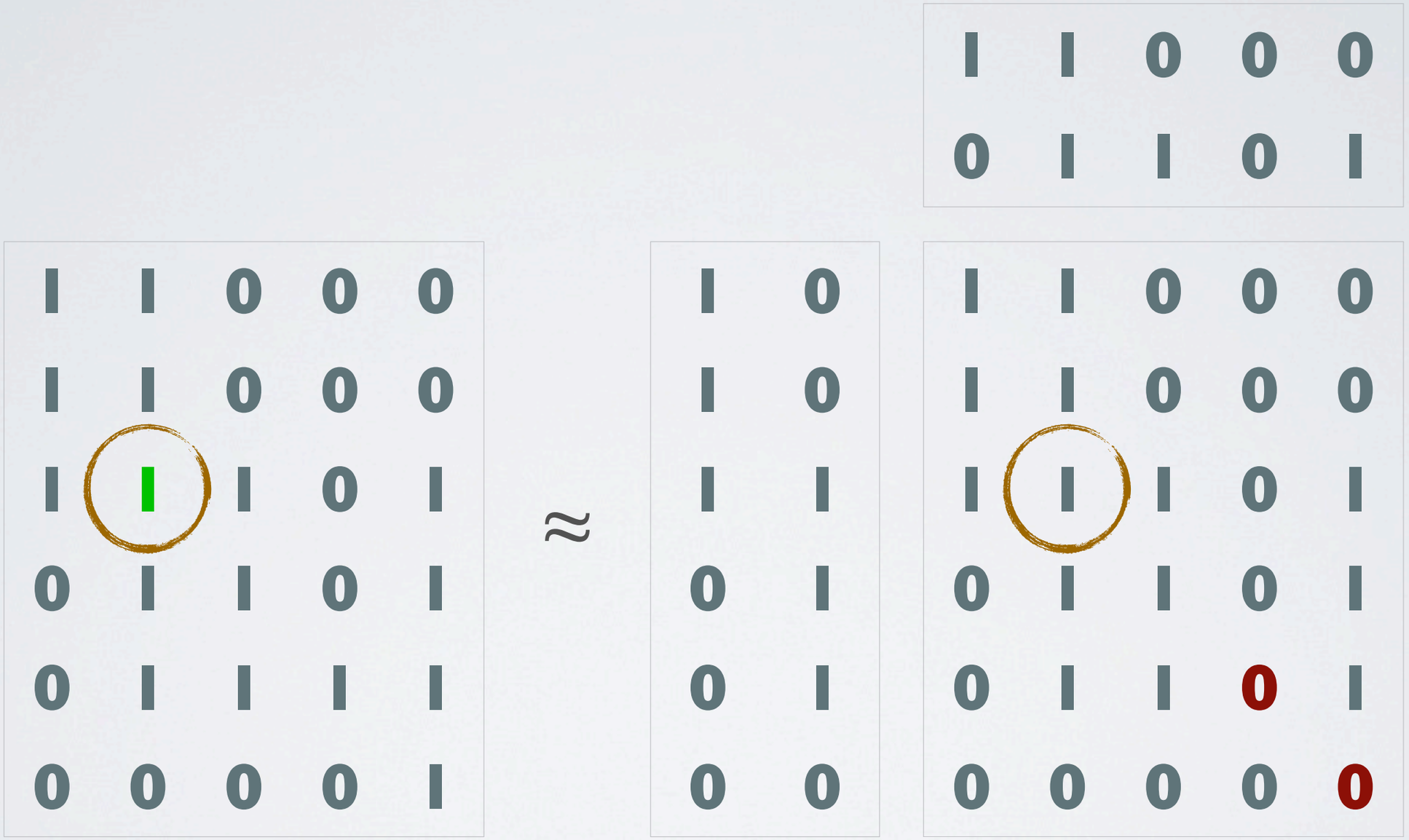
CASE 1: PREDICTED 1



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CASE 2: NO PRIOR INFO

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CASE 2: NO PRIOR INFO

1	1	0	0	0
0	1	1	0	1

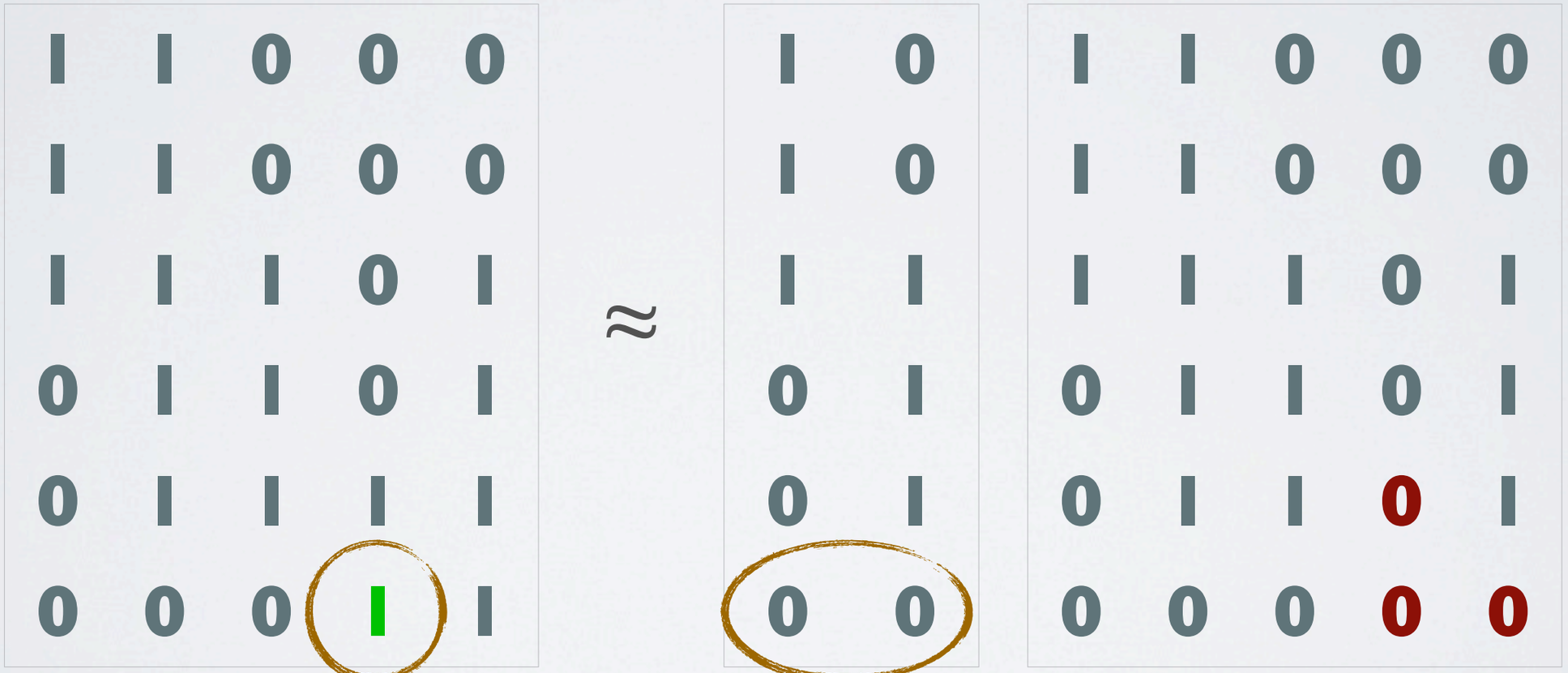
1	1	0	0	0
1	1	0	0	0
1	1	1	0	1
0	1	1	0	1
0	1	1	1	1
0	0	0	1	1

≈

1	0
1	1
0	1
0	1
0	0

1	1	0	0	0
1	1	1	0	1
0	1	1	0	1
0	1	1	0	1
0	0	0	0	0

CASE 2: NO PRIOR INFO



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CASE 3: PARTIAL INFO

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1	1	0	0	0
0	1	1	0	1

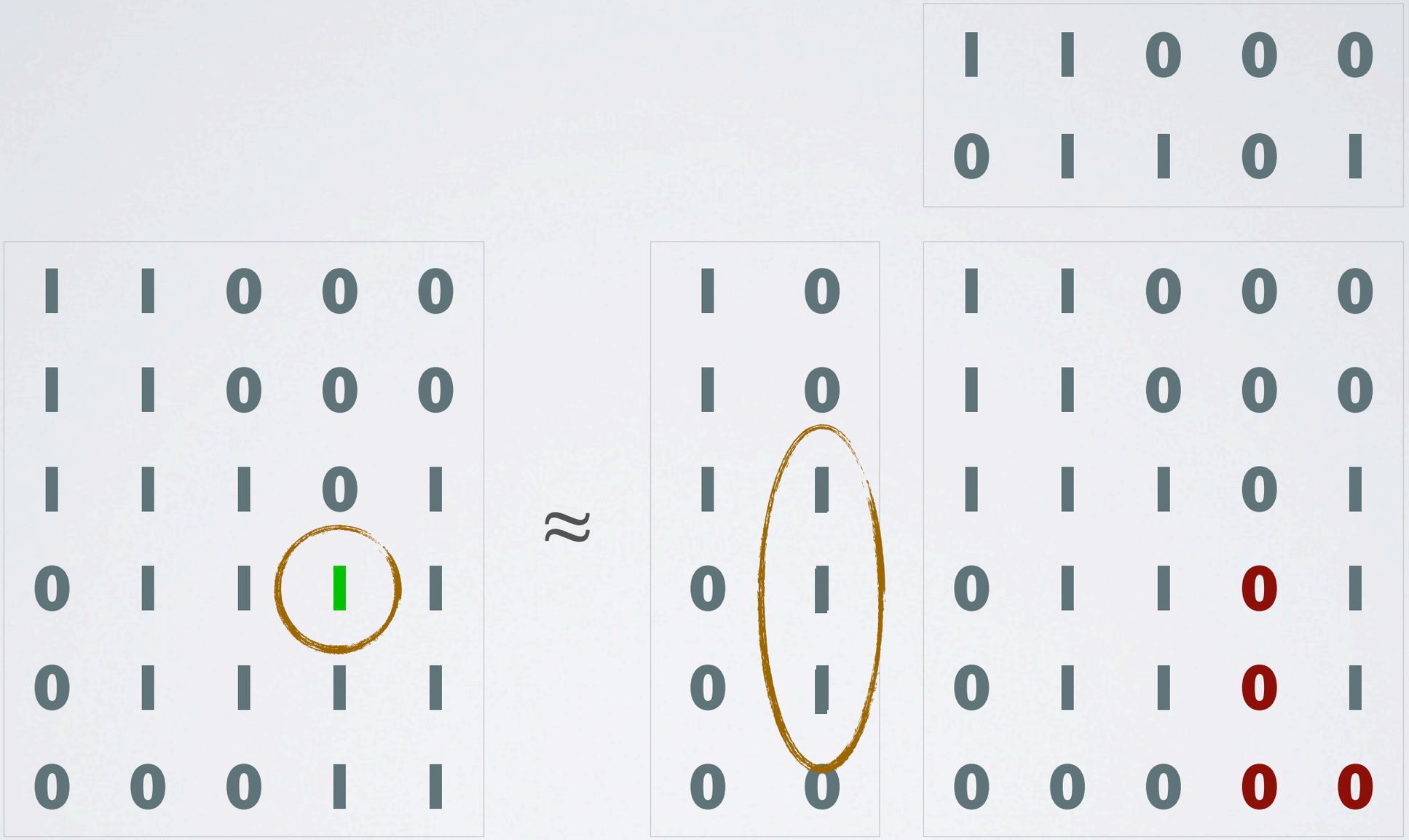
1	1	0	0	0
1	1	0	0	0
1	1	1	0	1
0	1	1	1	1
0	1	1	1	1
0	0	0	1	1

≈

1	0
1	1
0	1
0	1
0	0

1	1	0	0	0
1	1	1	0	1
0	1	1	0	1
0	1	1	0	1
0	0	0	0	0

CASE 3: PARTIAL INFO



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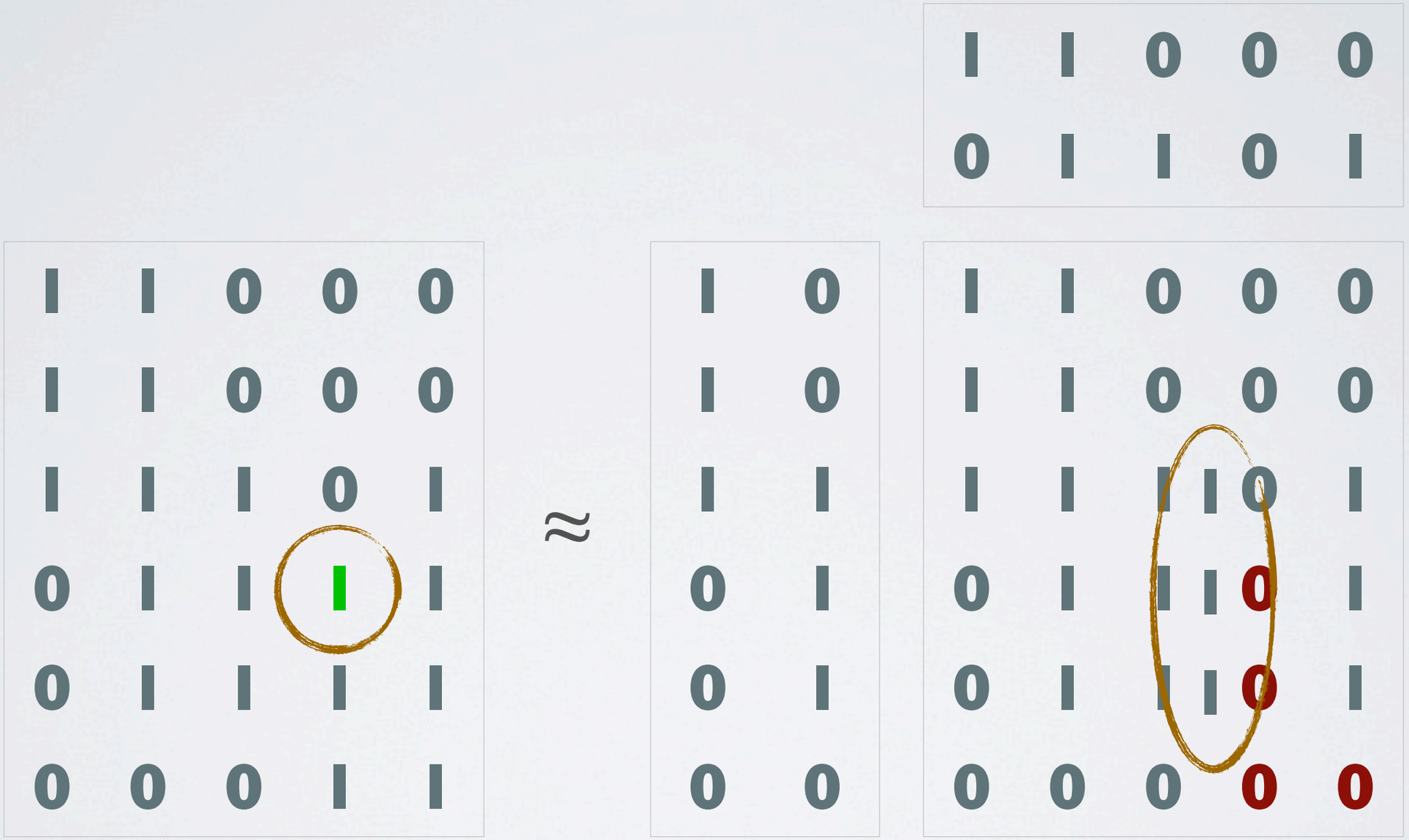
CASE 3: PARTIAL INFO



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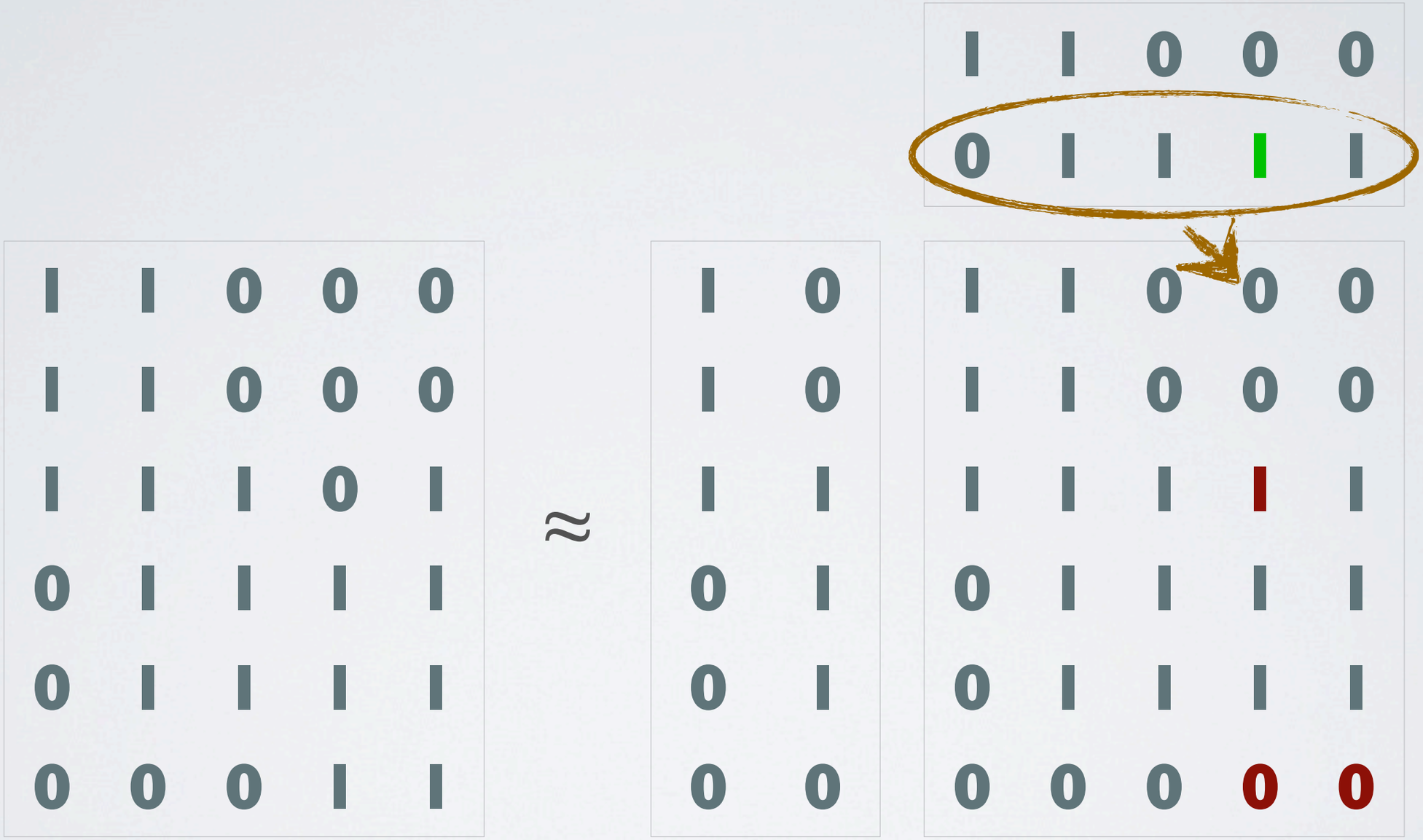

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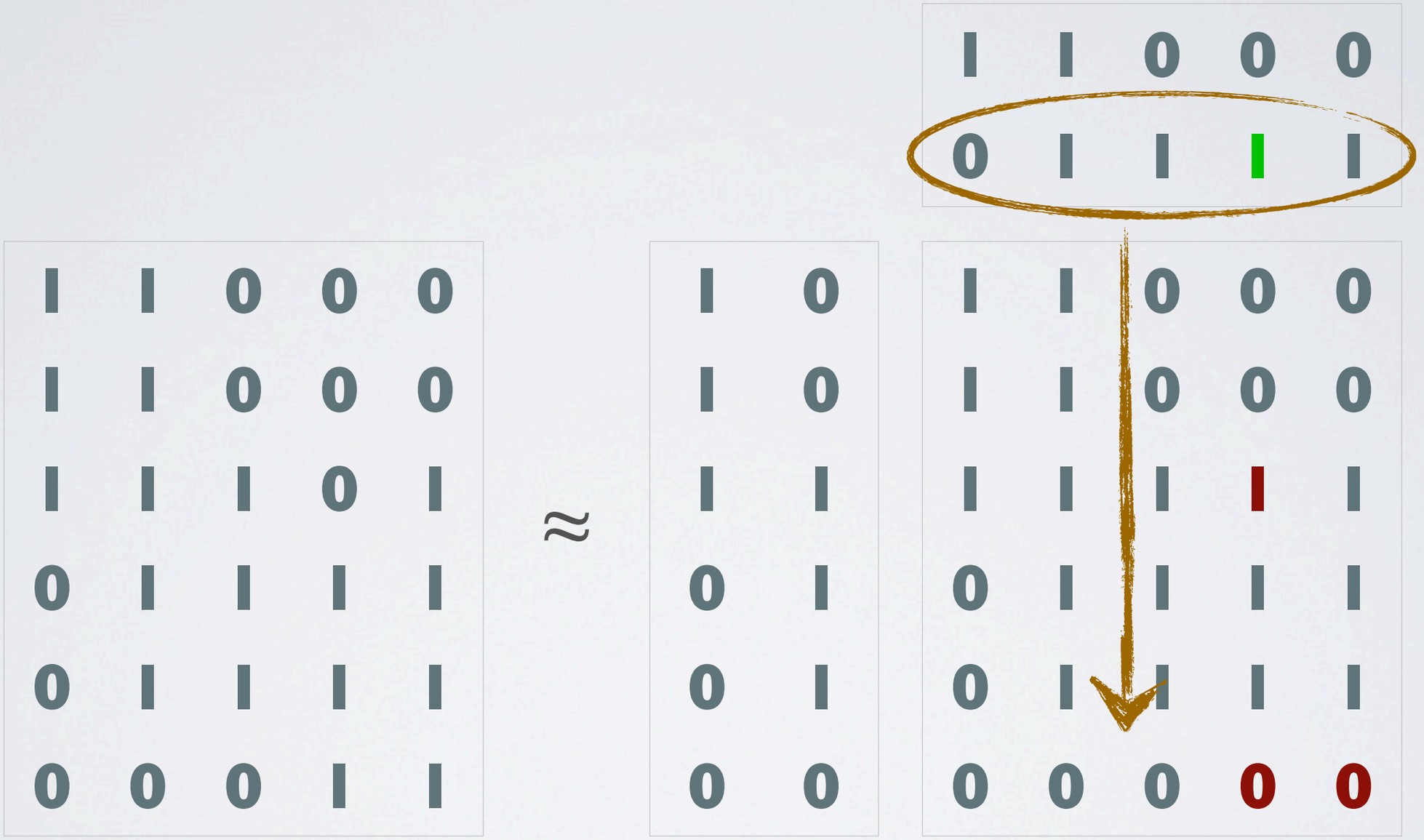


CASE 3: PARTIAL INFO

MAKING UPDATES FAST

- Recognizing the case is $O(k)$ with proper index structure
- Selecting the factor in Case 3 is worst-case $O(|\boldsymbol{B}| + |\boldsymbol{C}|)$
- Extending the factor can be very costly
  - Computing the fit for every row/column
  - We store the historical fits and take an optimistic approach on how much it could have improved
    ⇒We only need to consider those rows/columns where the factor could give a good fit

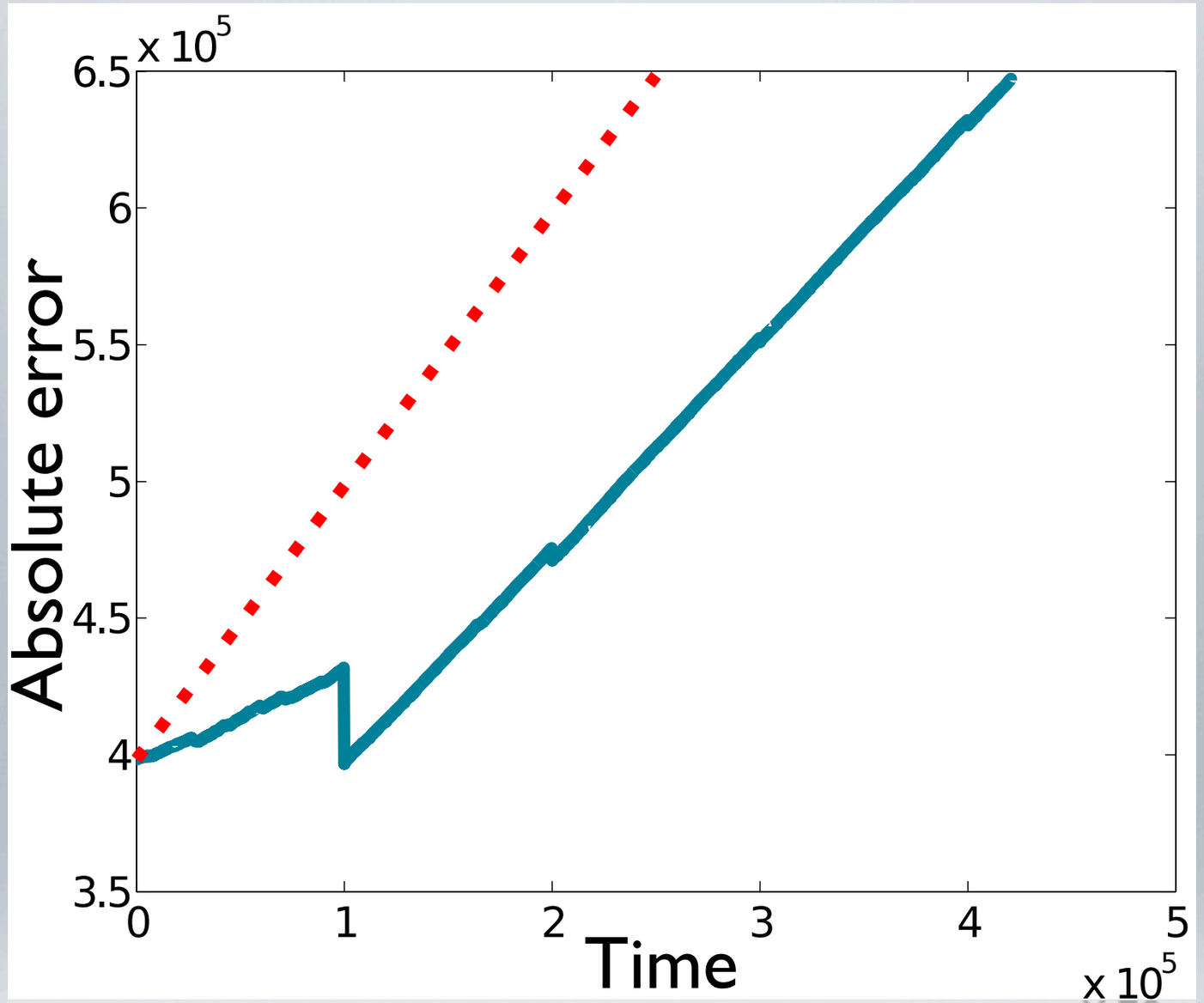
NON-ONLINE ALGORITHM

- Iteratively update ***B*** and ***C*** to remove 1s from them

- Fix ***B***, update ***C***; fix ***C***, update ***B***; etc.

- The problem is (still) NP-hard – we use a heuristic

- Computationally expensive



ERROR OVER TIME

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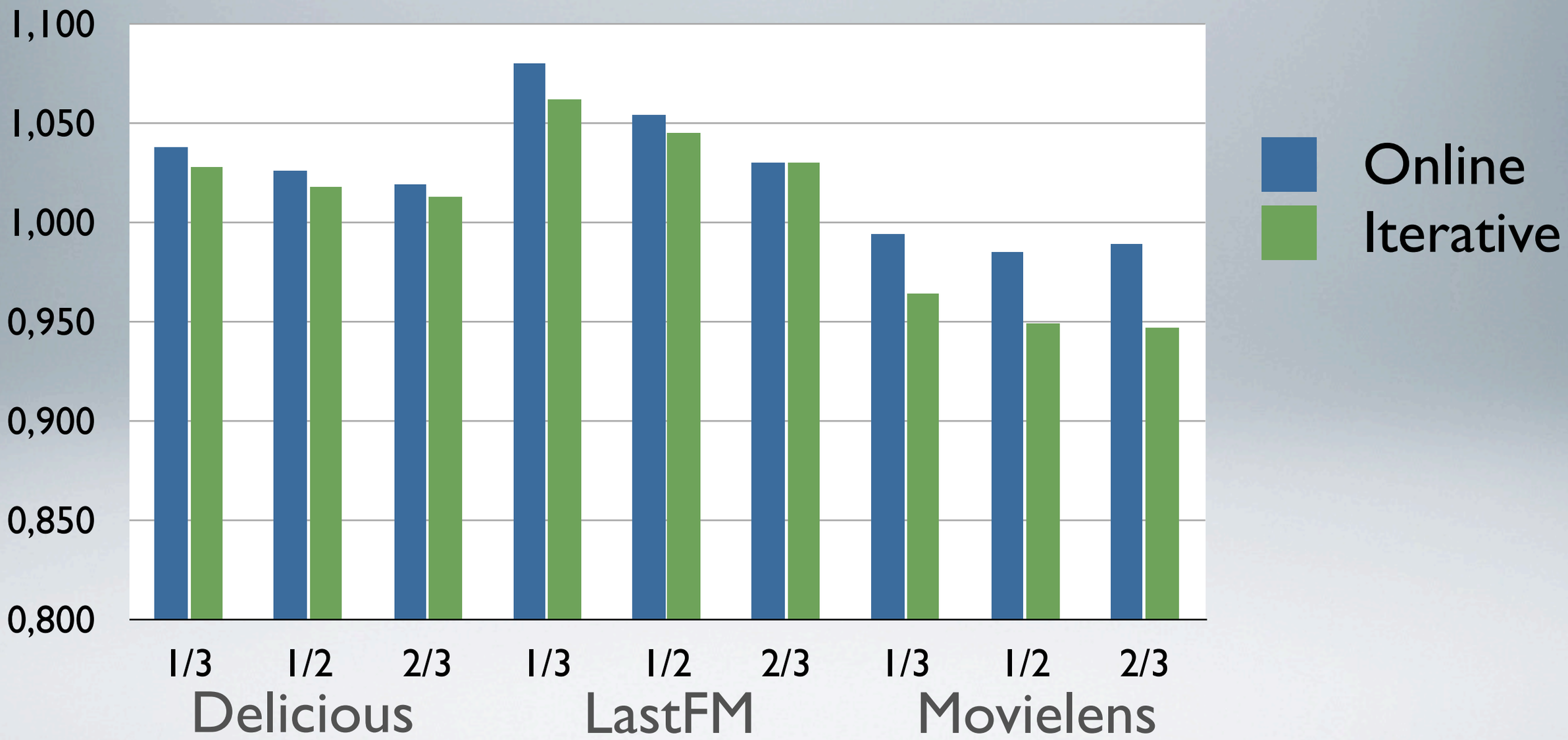
COMPETITIVE FACTOR



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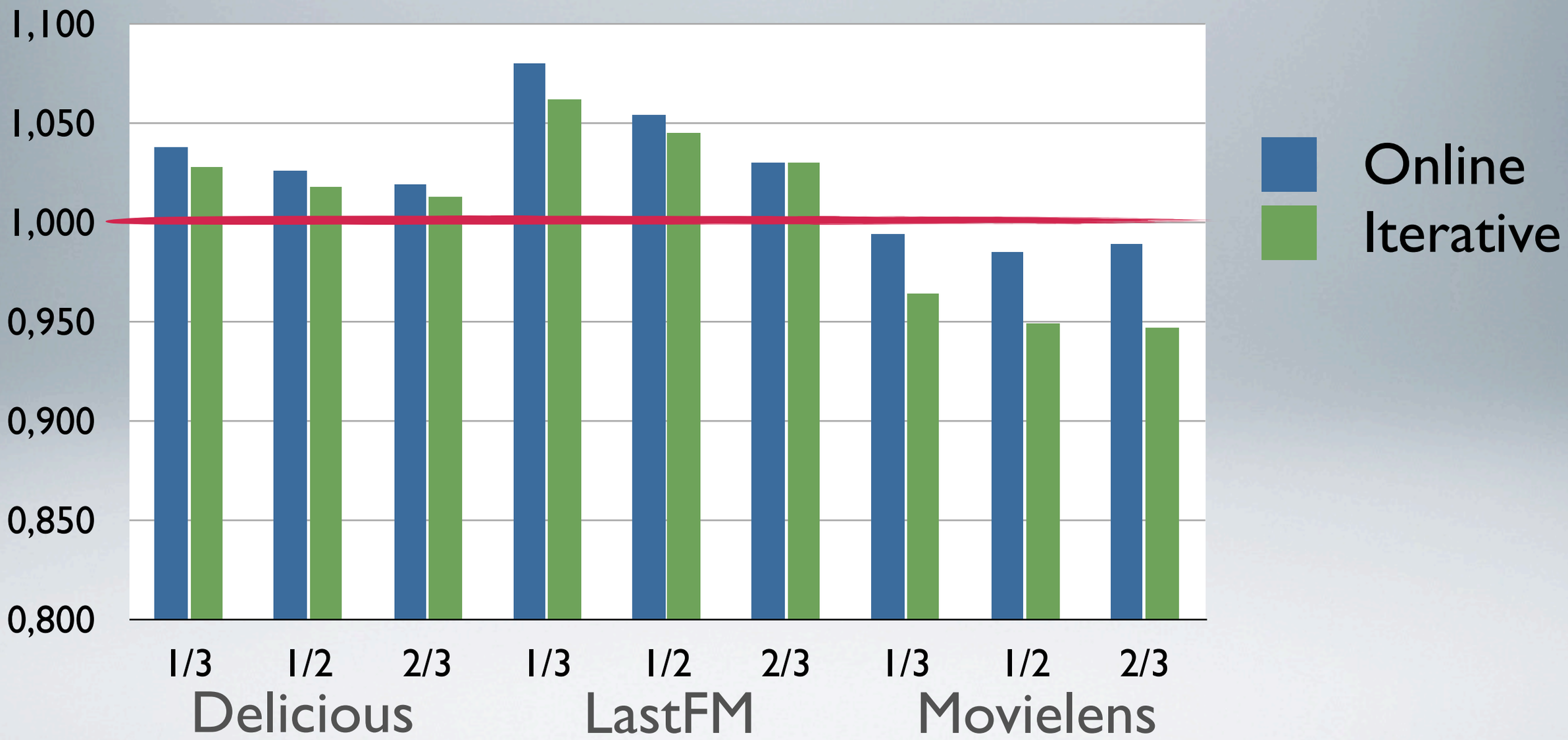
COMPETITIVE FACTOR



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TIME COMPLEXITY

	Sequence length			
Online?	561K	420K	281K	All
Yes	95	64	33	97
No	1045	1111	1097	

FUTURE WORK

- Adjusting the rank
- Better data structures and analysis
- Paralellization
- Tweaking the base algorithm for better prediction

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CONCLUSIONS

- Dynamically updating a Boolean matrix factorization is possible
- Simple idea performs very well and is reasonably fast
  - Can be better *and* faster than running the off-line algorithm

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Thank You!