Analysis IV
Spring 2011
Exercises 1

(1) Let $\{x_1, \ldots, x_n\}$ be a linearly independent set of vectors in the space $\mathbb{R}^n$. Prove that every $x \in \mathbb{R}^n$ has at most one representation of the form $x = \lambda_1 x_1 + \cdots + \lambda_n x_n$.

(2) Prove the linear subspace test (Theorem 1.3).

(3) Show that the set $F(S, V)$ in Definition 1.5 equipped with addition and scalar multiplication is a vector space.

(4) Prove that if S is the set of integers $\{1, \ldots, k\}$, then the set $F(S, \mathbb{F})$ can be identified with the space $\mathbb{F}^k$.

(5) Show that the mapping $\mathbb{F} : \mathbb{R}^3 \to \mathbb{R}^2$,
$$F(x, y, z) = (y + z, x),$$
is linear, that is, that $F \in L(\mathbb{R}^3, \mathbb{R}^2)$.

(6) Prove the Schwarz inequality: If $a_j, b_j \in \mathbb{F}$ for all $j = 1, \ldots, k$, then
$$\left(\sum_{j=1}^{k} |a_j||b_j| \right)^2 \leq \sum_{j=1}^{k} |a_j|^2 \cdot \sum_{j=1}^{k} |b_j|^2.$$