Analysis IV
Spring 2011
Exercises 3

(1) Let (E, d) be a metric space and let $x \in E$, $A \subset E$. Define
$$d(x, A) = \inf_{y \in A} d(x, y).$$
Show that $\{x \mid d(x, A) = 0\} = \overline{A}$.

(2) Let X be an infinite set. Let $\mathbb{T}$ consist of $\emptyset$, X and all sets G such that $X \setminus G$ is a finite set. Prove that $(X, \mathbb{T})$ is a topological space.

(3) Let $A \subset \mathbb{R}^n$ be a set whose every point has a neighborhood which includes only a countable number of points of A. Prove that A is countable. (Hint: Lindelöf's covering theorem)

(4) Prove that a collection of disjoint open sets in $\mathbb{R}^n$ is either finite or countable.

(5) Let f be a continuous real function on a metric space X. Let $\mathbb{Z}(f)$ be the set of all $p \in X$ for which $f(p) = 0$. Prove that $\mathbb{Z}(f)$ is closed.