Analysis IV
Spring 2011
Exercises 4

(1) Let $0 < a < 1$, and assume it is known that the functions $f_n(x) = nx(1-x)^n$ converge to $f(x) = 0$ as $n \to \infty$. Is the convergence uniform on $[a, 1]$? What about $[0, 1]$?

(2) Prove: If $m^*(B) = 0$, then $m^*(A \cup B) = m^*(A)$.

(3) Prove Corollary 2.4: If $A \subset \mathbb{R}^n$ is countable, then $m^*(A) = 0$.

(4) Prove Theorem 2.6: Outer measure m^* is translation invariant, that is, if $a \in \mathbb{R}$, then $m^*(A + a) = m^*(A)$ for all $A \subset \mathbb{R}$.

(5) Let A be the set of rational numbers between 0 and 1, and let $\{I_n\}$ be a finite collection of open intervals covering A. Prove that

$$\sum_n l(I_n) \geq 1.$$