Analysis IV
Spring 2011
Exercises 7

(1) Show that if f is a measurable function and $[a, b]$ is a real interval, the set
$$\{x \in \mathbb{R} \mid f(x) \in [a, b]\}$$
is measurable.

(2) Prove Theorem 2.26 (b').

(3) Let f be a non-negative measurable function. Show that $\int f\,dm = 0$ implies $f = 0$ a.e. (almost everywhere).

(4) Let f be a measurable function. Show that if E is a measurable set and $m(E) = 0$, then $\int_E f\,dm = 0$.

Hint. Prove this first for simple functions. Then for $f \geq 0$ using Theorem 2.26(d). Then note that $f = f^+ - f^-$.

(5) Let $f : [0, 1] \to \mathbb{R}$ be defined as
$$f(x) = \begin{cases} 1, & x \in (\mathbb{R} \setminus \mathbb{Q}) \cap [0, 1] \\ 0, & x \in \mathbb{Q} \cap [0, 1]. \end{cases}$$
Calculate
$$\int_{[0,1]} f\,dm.$$

(6) Prove Lemma 3.1.

Hint. Consider the cases $b = 0$ and $b \neq 0$ separately. Notice that $g : [0, \infty[\to \mathbb{R}$,
$$g(t) = (1 - \lambda) + \lambda t - t^{\lambda}, \qquad 0 < \lambda < 1,$$
has its minimum at $t = 1$.

More things you already know : A function f is *non-negative* if $f \geq 0$. This is not the same as the function being *positive*, as that means $f > 0$. The same's true for *non-positive* ($f \leq 0$) and *negative* ($f < 0$) functions.