Analysis IV
Spring 2011
Exercises 9

(1) Prove Lemma 3.9.

(2) Prove Hölder's inequality for series (line (3.4) in the lectures).

(3) Prove Minkowski's inequality for series (line (3.5) in the lectures).

(4) Prove Lemma 3.15 for $p = \infty$.

(5) Let $1 \leq p < q < \infty$. Define f and g as $f : [0, 2\pi] \to \hat{\mathbb{R}}$ and $g : [0, 2\pi] \to \hat{\mathbb{R}}$,

$$f(\theta) = \theta^{-1/q} \qquad \text{and} \qquad g(\theta) = \theta^{-1/2q}.$$

Show that $f \in L^p[0, 2\pi]$, $f \notin L^q[0, 2\pi]$, $g \in L^q[0, 2\pi]$, and $g \notin L^\infty[0, 2\pi]$. (You can assume f and g to be measurable.)

(6) Let $f \in L^1$ and $g \in L^\infty$. Show that

$$\int |fg| \, dm \leq d_{L^1}(f, 0) d_{L^\infty}(g, 0).$$

We write $\hat{\mathbb{R}}$ as a short for $\mathbb{R} \cup \{\infty\} \cup \{-\infty\}$, the extended real interval. We write $L^p[0, 2\pi]$ for $L^p([0, 2\pi])$.