Analysis IV
Spring 2011
Exercises 12

(1) Prove the second example on p. 30 of the lectures: If $f, g \in L^2$, then $fg \in L^1$ and

$$< f, g >= \int fg \, dm$$

is an inner product.

(2) Prove the third example on p. 30 of the lectures: If $a = \{a_n\}, b = \{b_n\} \in \ell^2$, then $\{a_n b_n\} \in \ell^1$ and

$$< a, b >= \sum_{n=1}^{\infty} a_n b_n$$

is an inner product.

(3) Prove Lemma 5.6.

(4) Prove Lemma 5.7.

(5) Let $1 \le p < q < \infty$. Prove that $\ell^p \subset \ell^q$. (Hint: start with those $x \in \ell^p$ for which $\|x\|_p = 1$.)

The remainders of the course:
- Tuesday and Wednesday, 12–13.4., the lectures as usual
- Friday 15.4. : Ex. 12, the usual time and place
- Tuesday 19.4. : The last lecture.
- Wednesday 20.4. : Ex. 13, 10:15–12:00, **M303**
- Thursday 28.4. : The second exam, 7.45–10.00, M103; note the exam will *really* begin at 7.45.
- Friday 29.4. : "Ex. 14", the usual time and place, just the answers to the second exam, nothing to do beforehand