Analysis IV
Spring 2011
Exercises 13

(1) Let the angle between $x, y \in \ell^2$ be defined as

$$\theta(x,y) = \cos^{-1}\left(\frac{< x, y >}{\|x\|\|y\|}\right)$$

with the standard inner product and norm of ℓ^2.

Let $x = \{1/2^n\}_{n=1}^{\infty}$ and $y = \{C/3^n\}_{n=1}^{\infty}$, where $C \in \mathbb{R}$. For what value of C does $\theta(x,y) = \pi/3$? For which value of C does $\theta(x,y) = \pi/2$?

(2) What is the angle between x^2 and x? Use the standard inner product and norm of $L^2([0,1])$.

(3) Let $T : C_{\mathbb{R}}([0,1]) \to \mathbb{R}$ be defined by

$$T(f) = \int_0^1 f(x)\,dx.$$

Show that T is continuous.

(4) Let $h \in L^{\infty}([0,1])$. Show that $T : L^2([0,1]) \to L^2([0,1])$,

$$T(f) = hf,$$

is continuous.

(5) Show that if $(x_1, x_2, \ldots) \in \ell^2$, then

$$(0, 4x_1, x_2, 4x_3, x_4, 4x_5, x_6, \ldots) \in \ell^2.$$

Let $T : \ell^2 \to \ell^2$ be defined by

$$T(x_1, x_2, \ldots) = (0, 4x_1, x_2, 4x_3, x_4, 4x_5, x_6, \ldots).$$

Show that T is continuous.

The remains of the course:

- Tuesday 19.4. : The last lecture.
- Wednesday 20.4. : Ex. 13, 10:15–12:00, **M303**, note: not the usual place; also note: the last real exercises
- Thursday 28.4. : The second exam, 7.45–10.00, M103; note the exam will *really* begin at 7.45.
- Friday 29.4. : "Ex. 14", the usual time and place, just the answers to the second exam, nothing to do beforehand