Chapitre 1

Exercises about barycenters

Exercise 1. Let E be the "ordinary" space of geometry and let $(O, \vec{\imath}, \vec{\jmath}, \vec{k})$ be a frame E. We suppose that E is euclidean and that the frame $(O, \vec{\imath}, \vec{\jmath}, \vec{k})$ is orthonormal.

a) Let K be a point with coordinates $(2, -10, 3)$ and let $\overrightarrow{u}$ be the vector $\overrightarrow{u} = 3\vec{\imath} + \frac{12}{5}\vec{\jmath} + 2\vec{k}$. Write the equation of the plane P which contains K and is orthogonal to $\overrightarrow{u}$.

b) Let A, B and C be the points of intersection of P with the axis Ox, Oy and Oz. Compute the coordinates of the points A, B and C.

c) Compute the coordinates of the centroide of the triangle ABC (point common to the three medians).

d) Draw a picture showing the points K, A, B and C and the vector $\overrightarrow{u}$.

e) Check that the vectors $\overrightarrow{KA}$, $\overrightarrow{KB}$ and $\overrightarrow{KC}$ are orthogonal to $\overrightarrow{u}$.

Exercise 2. Let A, B and C be three points in a plane P with a frame $(\Omega, \vec{\imath}, \vec{\jmath})$. We suppose the coordinates of A, B and C are respectively $(2, 3)$, $(6, 4)$ and $(-6, -\frac{1}{2})$. Show that

$$\begin{vmatrix} 2 & 6 & -6 \\ 3 & 4 & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix} = 0$$

How can you deduce from this equality that the three points are colinear ? Let (x_U, y_U) and (x_V, y_V) be the coordinates of two distinct points. Show that the equation of the line UV may be written

$$\begin{vmatrix} x & x_U & x_V \\ y & y_U & y_V \\ 1 & 1 & 1 \end{vmatrix} = 0$$

Exercise 3. Let E be a real affine space of dimension 3 and let $(O, \vec{\imath}, \vec{\jmath}, \vec{k})$ be a frame E. We do not suppose E to be euclidean and thus the frame $(O, \vec{\imath}, \vec{\jmath}, \vec{k})$ is not orthonormal.

a) Let A, B and C be points in P with coordinates $(4, 0, 0)$, $(0, 5, 0)$ and $(0, 0, 6)$ respectively. Find the equation of the plane ABC.

b) Find the coordinates of the centroid G of the triangle ABC. Check that G belongs to the plane ABC.

Exercise 4. Let A, B, C and C be a quadrilateral in a plane P. Denote by A' the middle of the segment AB, by B' the middle of the segment BC, by C' the middle of the segment CD and by D' the middle of the segment DA. Show that $A'B'C'D'$ is a parallelogram. What happens if we take the four points in a 3-dimensional space ? in a n-dimensional space ?

Exercise 5. Let A and B be two points. Describe the set of points which are barycenter of A and B assigned with masses of same sign. Generalize to three points, n points, an infinite sequence of points.

Exercise 6. Let ABC be a triangle and M_0 a point of the line BC. The parallel to AB through M_0 cuts the line CA in the point M_1. The parallel to BC through M_1 cuts the line AB in the point M_2. The parallel to CA through M_2 cuts the line BC in the point M_3. The parallel to AB through M_3 cuts the line CA in the point M_4. And so on. Describe what is going to happen.

Exercise 7. Show the equivalences that are stated in the sketch 2 on page 11.

Exercise 8. Let $ABCD$ be a square in a usual euclidean plane. Find the normalized barycentric coordinates of the point D relatively to the triangle ABC.

Exercise 9. Let $ABCD$ be a parallelogram in a usual affine plane. Find the normalized barycentric coordinates of the point D relatively to the triangle ABC. Compare with the exercise 8.

Exercise 10. Let A, B and C be three points in a plane P and let $t \in [0, 1]$. Let $D(t)$ be the barycenter of A and B assigned with the masses $(1-t)$ and t ; let $E(t)$ be the barycenter of B and C assigned with the masses $(1-t)$ and t and let $F(t)$ be the barycenter of $D(t)$ and $E(t)$ assigned with the masses $(1-t)$ and t. Describe the curve $[0, 1] \to P, t \mapsto F(t)$.