Exercise sheet 2

Exercise 1. We start with a triangle ABC, for instance $A(2,4)$, $B(0,0)$ and $C(6,0)$. Let P be the point on the line AB such that A is between B and P and such that $AP = \frac{1}{2}BA$ (in the above instance $P(3,6)$).

Let Q be the point on the line CA such that A is between C and Q and such that $AQ = AP$.

Let R be the point on the line CB such that B is between C and R and such that $CR = CQ$.

Let S be the point on the line AB such that B is between A and S and such that $BS = BR$.

Let T be the point on the line AC such that C is between A and T and such that $AT = AS$.

Let U be the point on the line BC such that C is between B and U and such that $CU = CT$.

a) Show that $BU = BP$.

b) Draw the arcs $\overset{\frown}{PQ}$ with center A, $\overset{\frown}{QR}$ with center C, $\overset{\frown}{RS}$ with center B, $\overset{\frown}{ST}$ with center A, $\overset{\frown}{TU}$ with center C, $\overset{\frown}{UP}$ with center B. Let D be the domain inside the curve $\overset{\frown}{PQRSTUP}$. Study the 2-periodic trajectories of D.

Exercise 2. a) Draw the parallelogram $OPQR$ following the instructions below :

- Start with the segment OI where $O(0,0)$, $I(10,0)$.
- Draw the lines OJ and IJ meeting at J such that $\widehat{IOJ} = 15°$ and $OIJ = 105°$.
- Draw the line JP such that O, I and P are on a line and $\widehat{IJP} = 60°$.
- Construct the point Q on the line PJ such that $\widehat{JIQ} = 30°$ and J is between P and Q.
- Construct the point R such that $OPQR$ is a parallelogram.
- Construct the point K on the segment RQ such that $RK = PI$.
- Construct the point L on the segment RO such that $RL = PJ$.

Let D be the domain inside the parallelogram $OPQR$.

b) Construct a trajectory t in D starting from $M(7,0)$ in the direction $\vec{v}$ such that $\alpha = (\overrightarrow{MP}, \vec{v}) = 75°$.

c) Are they any periodic trajectories in D ?

d) Is the trajectory t periodic ?

e) Is the trajectory t ergodic ?