

# PREPARING FOR THE PROOF OF PYTHAGORAS' THEOREM BY EUCLID

## Euclid's Elements. Book 1

### Preparing Geogebra

- a) In the menu "Options" or "Vaihtoehdot" choose "Kieli" and then "English(UK)".
- b) In the menu "Options" — "Labelling", check "New points only".
- c) In the menu "View" uncheck "Axes", verify that "Algebra window" is checked.

### Exercise 1.

Choose two points  $A$  and  $B$ . Use the tool "regular polygon" to draw squares :  $ABCD$ ,  $ADEF$ ,  $AFGH$ ,  $AHIJ$  where  $J = B$  and  $BDKL$  where  $K = H$  and  $H = L$ .

The sketch you have got can be thought of as an illustration of  $AC = \sqrt{2} AB$ .  
Clear the plane.

### Exercise 2.

- a) Choose three points  $A$ ,  $B$  and  $C$ .
- b) Draw the line  $a$  through  $A$  and  $B$ .
- c) Draw the line  $b$  parallel to  $a$  going through  $C$ .
- d) Choose two points  $D$  and  $E$  on  $b$  and one point  $F$  on  $a$ .
- e) Using the tools "parallel line" and "intersecting point" construct the point  $G$  such that  $DEGF$  is a parallelogram.
- f) Use the tool "Polygon" to draw the surface of the parallelogram  $DEGF$  and get its area (in some unknown unit) in the algebra window.
- g) Choose an other point  $H$  on the line  $a$  and construct the parallelogram  $DEIH$ . Check that  $DEGF$  and  $DEIH$  have the same area.
- h) Let the point  $F$  move on  $a$  and check that the area of  $DEGF$  does not change.
- i) Put the points  $F$ ,  $G$ ,  $H$  and  $I$  on the line  $a$  in the order  $FGHI$ . Call  $J$  the intersection of  $DH$  and  $EG$ . Note the areas of the parallelograms  $DEGF$  and  $DEIH$  and of the triangles  $DHF$ ,  $GEI$ ,  $GJH$  and  $DEJ$ . Check that:

$$\begin{aligned}\text{area } DEGF &= \text{area } DHF - \text{area } GJH + \text{area } DEJ \\ &= \text{area } EIG - \text{area } GJH + \text{area } DEJ = \text{area } DEIH\end{aligned}$$

### Exercise 3.

- a) Construct a triangle  $ABC$  with a right angle at  $A$ .
- b) Construct three squares outside the triangle  $ABC$ . Let these squares be  $BADE$ ,  $ACFG$  and  $CBHI$ .
- c) Draw the segments  $EC$  and  $AH$ . Why are the triangles  $BEC$  and  $BAH$  equal ?
- d) Draw the parallelogram  $EBCJ$ . Why is the area of the triangle  $BEC$  half the area of the square  $BADE$ ?
- e) Draw through  $A$  the parallel to  $BH$  and let it intersect  $HI$  in the point  $K$ .
- f) Construct the parallelogram  $ABHL$ . Why is the area of  $ABHL$  equal to that of the square  $BADE$ ?
- g) Let  $M$  be the intersection point of  $AK$  and  $BC$ . Why is the area of  $BHKM$  equal to that of the square  $BADE$ ?
- h) Show Pythagoras' theorem and use colors to illustrate the proof.