"DO IT YOURSELF"

Ordered constructions of Euclid's Elements. Book 1

Preparing Geogebra

a) In the menu "Options" or "Vaihtoehdot" choose "Kieli" and then "English(UK)".
b) In the menu "Options" — "Labelling", check "New points only".
c) In the menu "View" uncheck "Axes", verify that "Algebra window" is checked.
d) In the menu "Tools" — "Customise toolbar"

remove from toolbar :

- "New Point"
- "Line through two points" and "Polygone"
- "Perpendicular line"
- "Circle with center through point"
- "Angle"
- "Mirror object at point" and/or "Mirror object at line"
- "Insert text" and "Slider"

construct following toolbar :

- "Move"
- Separation
- "New Point"
- Separation
- "Intersect two objects"
- Separation
- "Segment between two points"
- Separation
- "Ray through two points"
- Separation
- "Circle with center through point"
- Separation
- "Move drawing pad"
- Separation

end by clicking "Apply".

Proposition α'. *To construct an equilateral triangle with a given basis.*

We are creating a tool that draws an equilateral triangle on a given side AB(more precisely : given the two endpoints A and B)

1) Create two points, called A and B.
2) Draw the segment AB.
3) Draw the circle c with center A and radius AB and the circle d with center B and radius BA. The letters c and d are not on the picture but in the algebraic window. Choose the intersection point C such that the triangle ABC is direct (direct in the meaning of mathematicians, that is counterclockwise).
4) Draw the segments BC called b and the segment CA called e.
5) In the menu "Tools", choose "Create new tool"

As Output objects choose
- C
- a (that is segmentAB)
- b (that is segmentBC)
- e (that is segmentCA)

Click "Next" or "Input objects"
As Input objects the choice si allready made for you : A and B.
Click "Next".
Name the tool "EquilateralTriangle"
Finish. Success : OK.

6) Use the menu "Move" to select your drawings and delete all.
7) Try your new tool several times. Delete all.
8) Draw a regular hexagone with this new tool.
9) Delete all.

Proposition β'
At a given point, to place a straight line equal to a given straight line.
We are creating a tool such that given three points A, B and C, it draws a sement AF with one endpoint at A and with the same length as BC ("equal" in the sense of Euclid).
1) Choose three points A, B and C.
2) Draw BC
3) Use the tool "EquilateralTriangle" to construct an equilateral triangle with basis AB.
4) Construct the two rays (or half-lines) with endpoint D and going through A and B.
5) Draw the circle with ceqnter B and radius BC.
6) Take the intersection point E common to the circle and the ray DB.
7) Draw the circle with center D and radius DE.
6) Take the intersection point F common to the last circle and the ray DA.
7) Draw the segment AF, called i. Chek in the algebraic window that it has the same length as BC.
8) Create a new tool.
- Output objescts : point F, segment i $(= AF)$.
- Input objects : point A, point B, point C.
- Name : "SegmentAtPointGivenLength"

9) Clear your page and try your new tool.
10) Delete all.

Exercise 1. Proposition γ'. *Given two unequal straight lines, to cut off from the greater a straight line equal to the less.*
Create a tool such that given four points A, B, C and D, it draws, when $AB > CD$, a sement AE with one endpoint at A and the other E on AB, with the same length as BC and call it "CutOffASegmentFromAGreater". The only tools to be used are the four from the initial toolbar and the two new tools we have built. *Hint* : Begin by building a segment AE "equal" to CD.
Warning : your tool works only if the four points are distinct. Remember that in following exercises. You might do a special tool with 3 points as input that cuts off a segment AC from a greater one AB, where these two segments have a common end A.

Exercise 2. Proposition θ'**.** *To bisect a given rectilinear angle.*

Create a tool such that given two rays AB and AC with common endpoint A, it creates a ray c with endpoint A such that $\widehat{BAc} = \widehat{cAC}$. Call your new tool "BisectAnAngle".

Be careful in which order you take the points if you want to get the good ray and not the one opposite to the one you would want.

Exercise 3. Proposition ι'**.** *To bisect a given finite straight line.*

Follow the book of Euclid to create a tool such that, given a segment AB, it creates a point D on that segment and such that $AD = DB$. *Hint* : Use the tools "EquilateralTriangle" and "BisectAnAngle".

Exercise 4. Proposition $\iota\alpha'$**.** *To draw a straight line at right angles to a given straight line from a given point on it.*

Follow the book of Euclid to create a tool such that given a segment AB and a point C on that segment it draws a point F and a segment CF perpendicular to AB.

Exercise 5. Proposition $\iota\beta'$**.** *To draw a straight line perpendicular to a given infinite straight line from a given point not on it.*

Follow the book of Euclid to create a tool such that given a line AB and a point C not on that line, it draws a point F and a segment CF perpendicular to AB.

Exercise 6. Proposition $\kappa\beta'$**.** *To construct a triangle out of three straight lines which equal three given straight lines : thus it is necessary that the sum of any two of the straight lines should be greater than the remaining one.*

Follow the book of Euclid to create a tool such that given three segments AB, CD and EF and a ray GH it creates a triangle GIJ such that I lies on GH, $GI = AB$, $IJ = CD$ and $JG = EF$.

Exercise 7. Proposition $\kappa\gamma'$**.** *To construct a rectilinear angle equal to a given rectilinear angle on a given straight line and at a point on it.*

Proposition $\lambda\alpha'$**.** *To draw a straight line through a given point parallel to a given straight line.*

Proposition $\mu\beta'$**.** *To construct a parallelogram equal to a given triangle in a given rectilinear angle.*

Proposition $\mu\delta'$**.** *To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.*

Proposition $\mu\varepsilon'$**.** *To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.*

Proposition $\mu\varsigma'$**.** *To describe a square on a given straight line.*

Construct the corresponding tools.