Exercises

The exercises are of different difficulty, just choose among them and make your brain warm some time with them.

About rational tangles (Conway's dance) :

- show that if you compute $-1/x$ by exchanging numerator and denominator and changing sign, and $x+1$ by adding the denominator to the numerator, then all fractions obtained by dancing following Conway's dance steps are irreducible (you'll never get 35/55)
- given any rational number obtained after some steps of Conway's dance, find an algorithmic way to go down to zero by using only the allowed steps ($x \longmapsto x+1$ and $x \longmapsto -\frac{1}{x}$) (so that if we trust Conway's theorem, we can always untangle by dancing). Can you prove your algorithm (in particular that it terminates in a finite number of steps) ?
- Conversely, can you find the moves that will construct the rational tangle associated with a given rational number ? For instance give a sequence of moves (write it as a word like PKPPPKPKPPKPP) that will construct the number $\frac{89}{57}$ (if you send it to me by mail, I can check on my computer).
- Since applying twice the transformation $x \longmapsto -\frac{1}{x}$ is the identity on rational numbers, Conway's theorem suggests that two rotations of 90°, that is one half turn, doesn't change the tangle. True or false ?
- Start the dance with the "untangle" as usual and turn (käännä ympäri) once. What happens if you then twist once ? twice ? three times ? Can you explain what is the "rational numbers" counterpart (or point of view) of this behaviour ?

About knots :

You've got a table of knots, with numbers (the number 6_3 for instance denoting the third knot having 6 crossings in the table).

- Can you guess which of these knots (give its number) the first group of 3 students obtained with their arms at the beginning of the lecture ?
- Is there any knot (at least one) for which you can prove that it is not the unknot ? which ones ?
- Show that the three red knots you got on paper sheet are equal ; which one of the table is it ?
- Solmu, punos, letti, takku : draw a diagram to show which term is more general, which is more particular compared to others.
- Show that the figure-eight knot (number 4_1 in the table) is amphicheiral, which means equal to its mirror image (use ropes, or give a sequence of Reidemesiter moves).
- What do you think is the minimal number of sides a polygonal knot should have in order to be different from the unknot ?

- If you make an ordinary shoe-lace knot on a rope, construct (if possible, otherwise show the impossibility) a second knot on the rope so that pulling both ends make all knots disappear (i.e. the second knot is cancelling the first one).
- Look at this video : http ://www.youtube.com/embed/eSKCi9ml4ME and describe everything you wouldn't have noticed before the lectures and that you can notice or even explain now using knot theory (arms only, arms + legs + floor, ...), for instance at 1 :50-1 :58, 2 :19-2 :25, 4 :18-4 :20, 4 :29, 5 :13-5 :17, ...

About borromean rings :

- How did we prove that any two of these rings form the unlink with 2 components ? Did we prove that all three components do not form the unlink with 3 components ?
- We have seen two pictures of borromean rings (it is a link with 3 components) : one given by three ellipses in three pairwise orthogonal planes, the other one by three "slightly wavy circles." Can a borromean link be made of three circles (necessarily in different planes) ?