Solmut

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What about knots ?

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- Braids : from elementary particles to mexican bracelets.
- More art : celtic links.

Conway's dance



and rational tangles.

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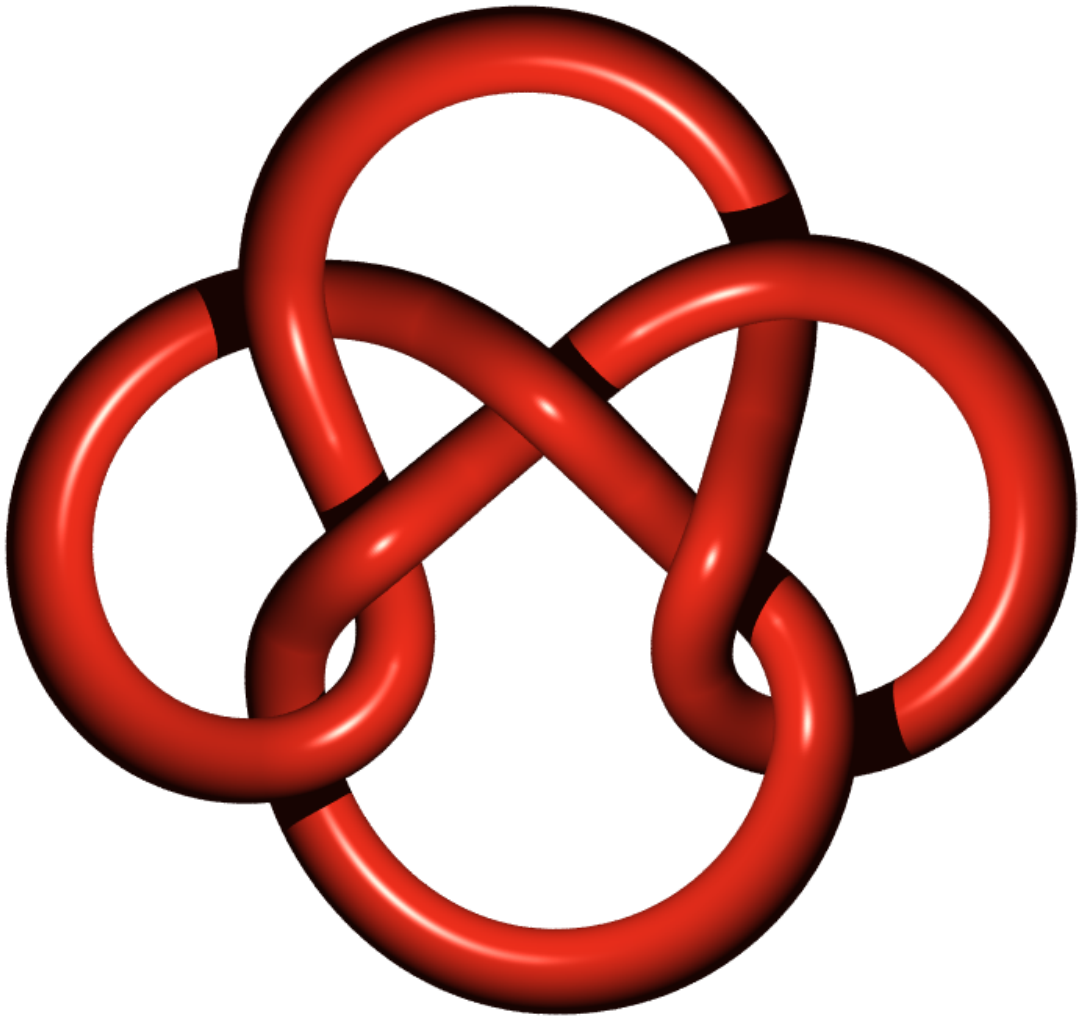
Theorem (Conway)

The operations $x \mapsto x+1$ *and* $x \mapsto -\frac{1}{x}$ *induce a one to one correspondance between rational numbers and rational tangles.*

A **knot (solmu)** is a simple closed curve in $\mathbb{R}^3$



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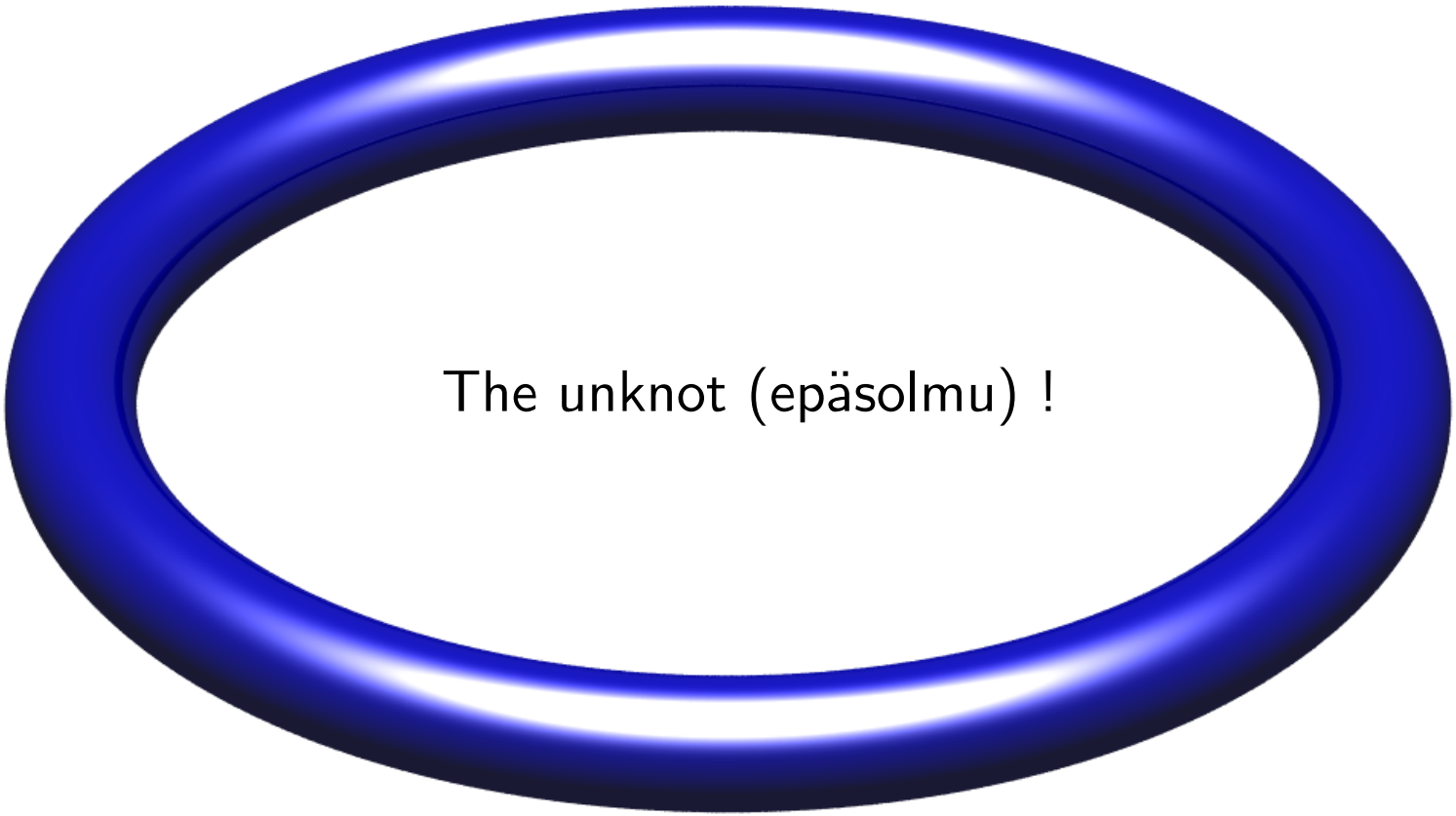
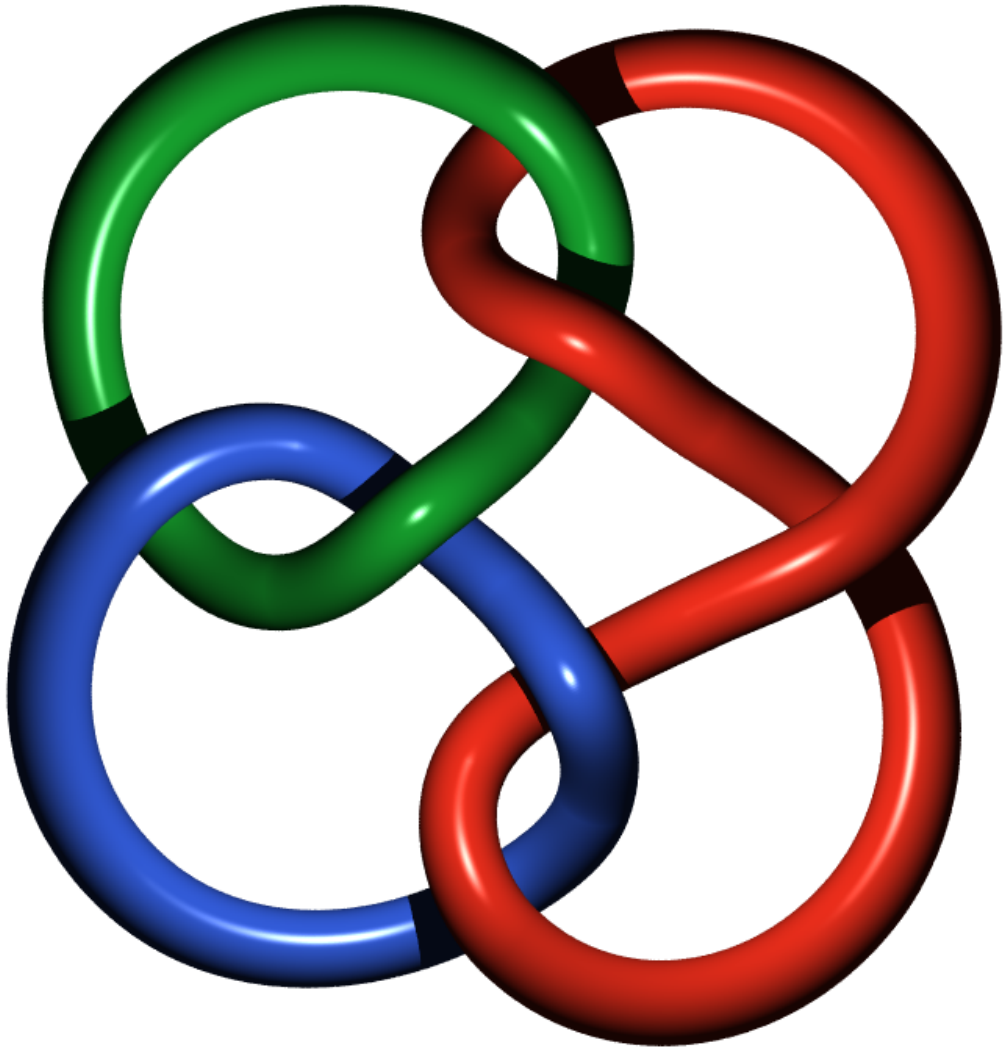


A **link (punos)** is a union of disjoint knots



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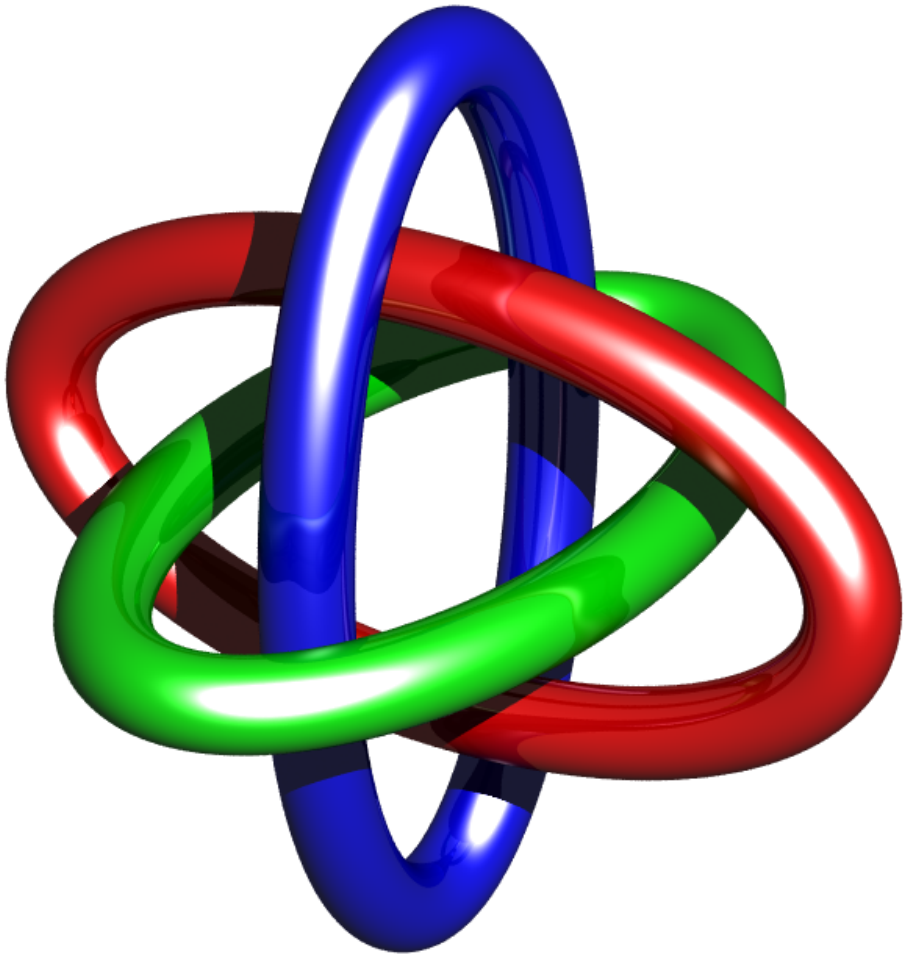

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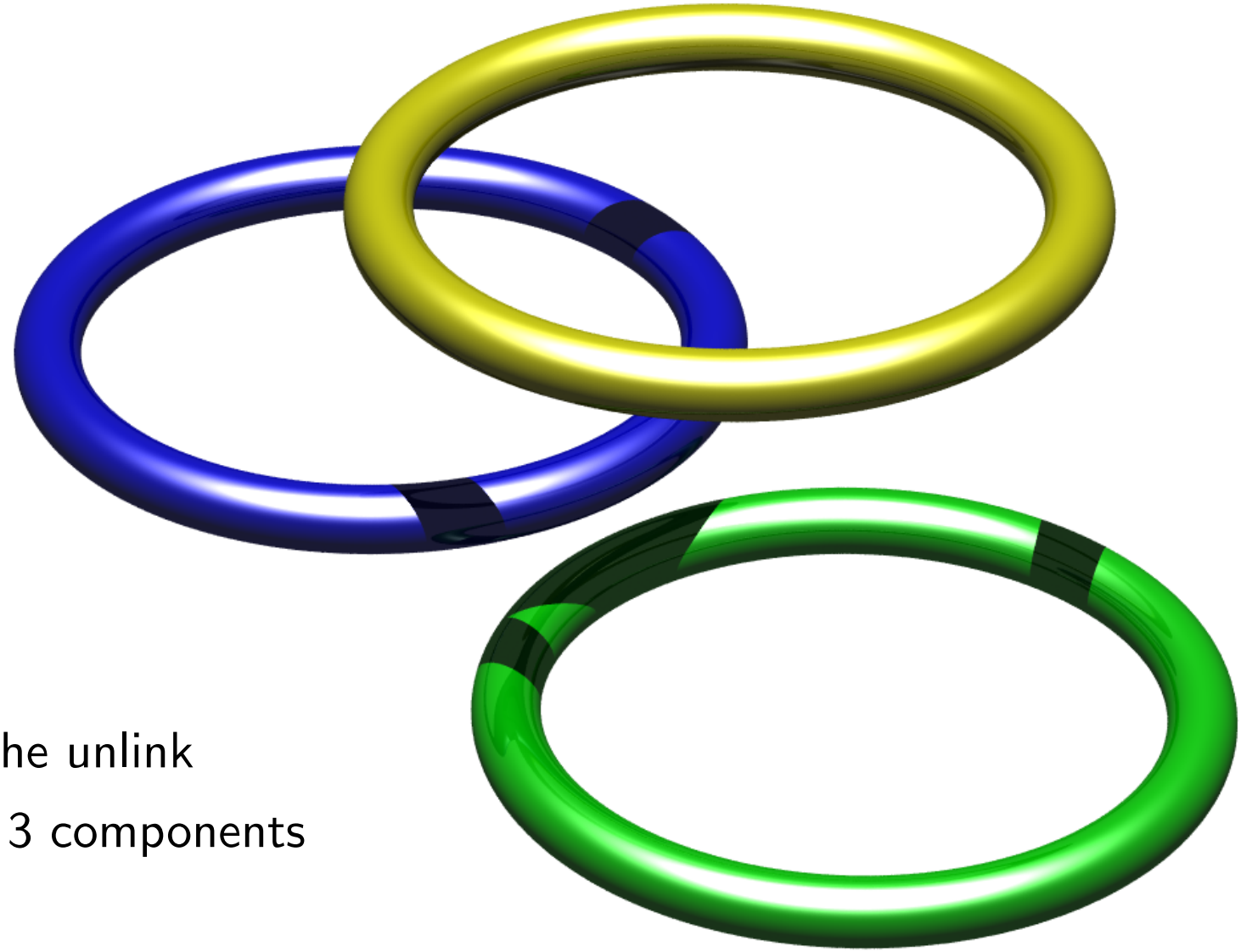
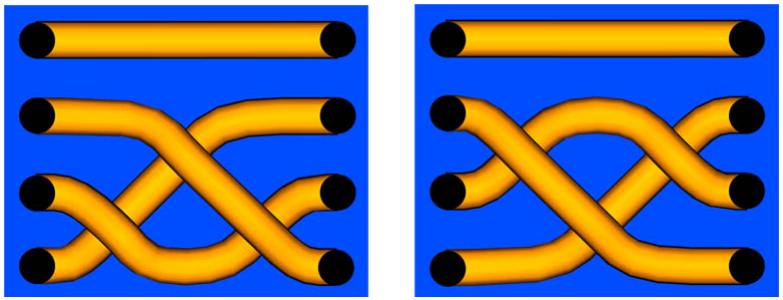


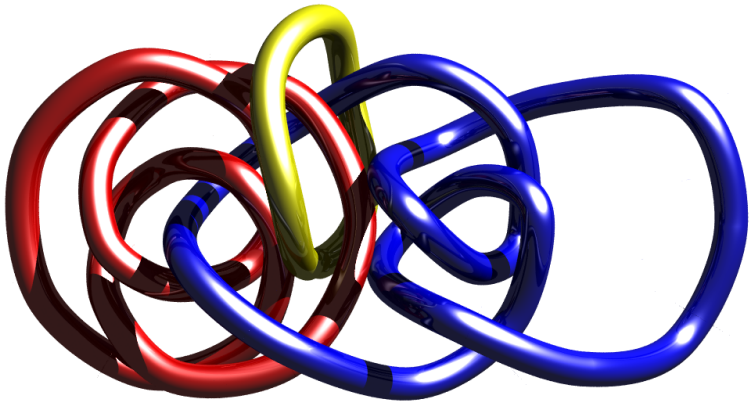
The unlink

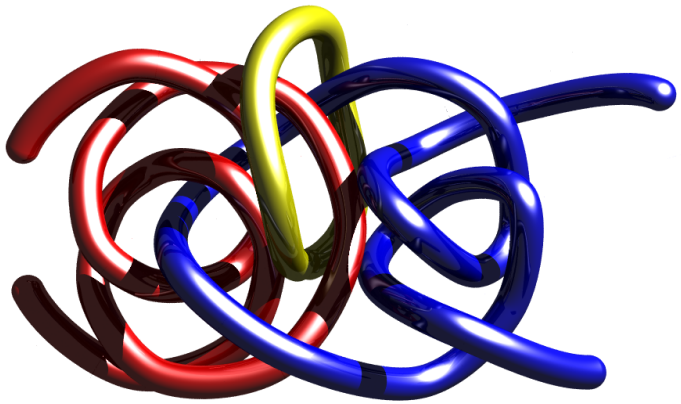
with 3 components

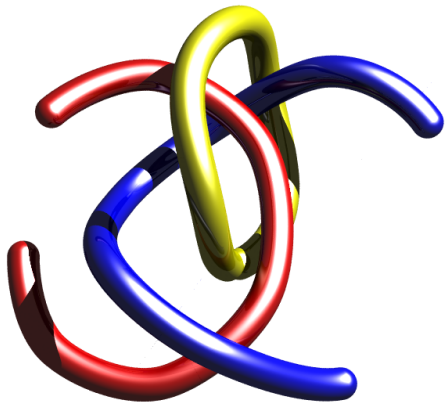
A **braid (letti tai palmikko)** is union of interweaved strands



A **tangle (takku)** is union of simple curves.

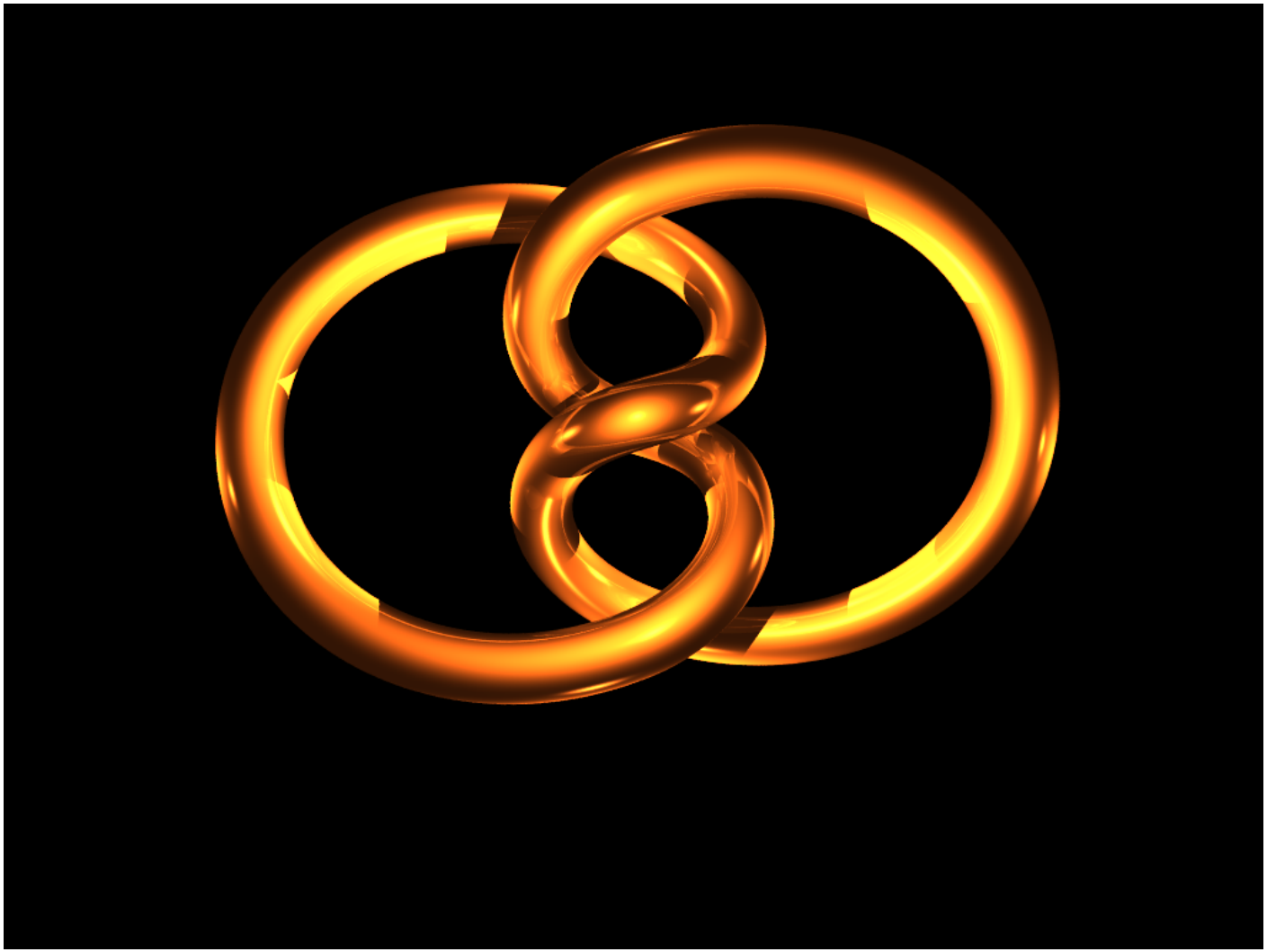






- **Knot (solmu)** = simple closed curve in space
- **Link (punos)** = union of disjoint knots
- **Braid (letti / palmikko)** = interweaved strands
- **Tangle (takku)** = union of simple curves

What is this ?



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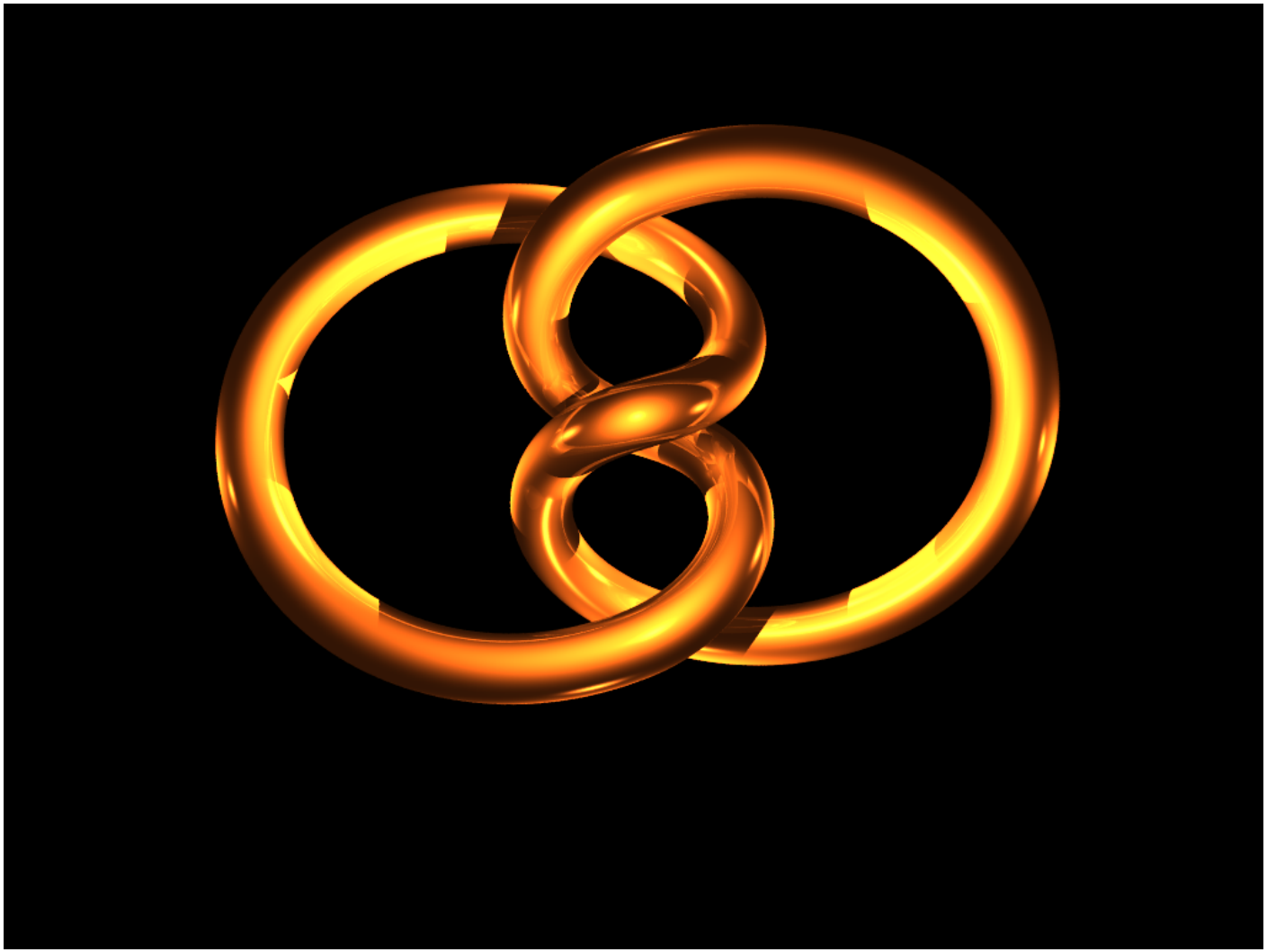


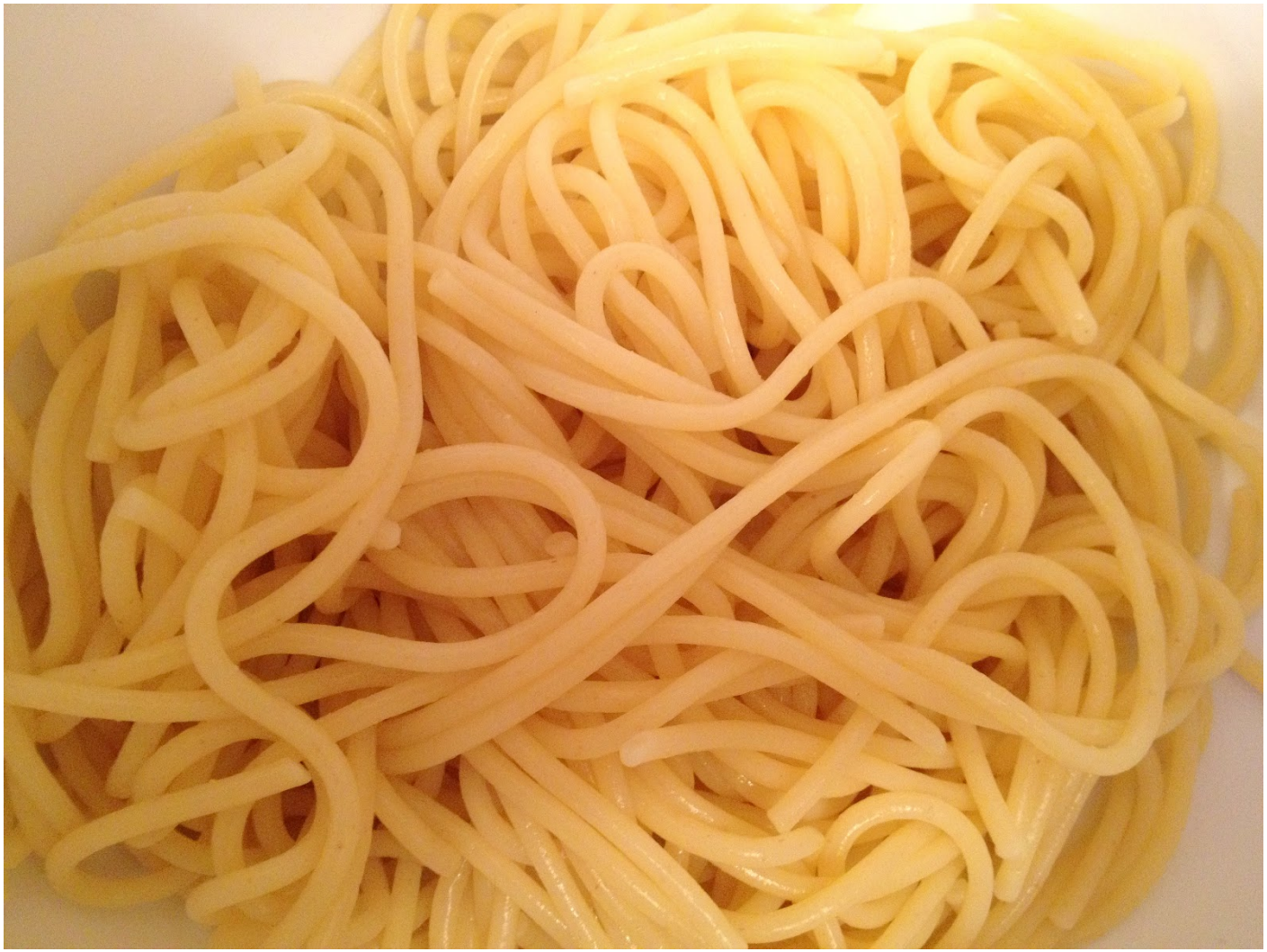
A knot

What is this ?

What is this ?

A braid

What is this ?



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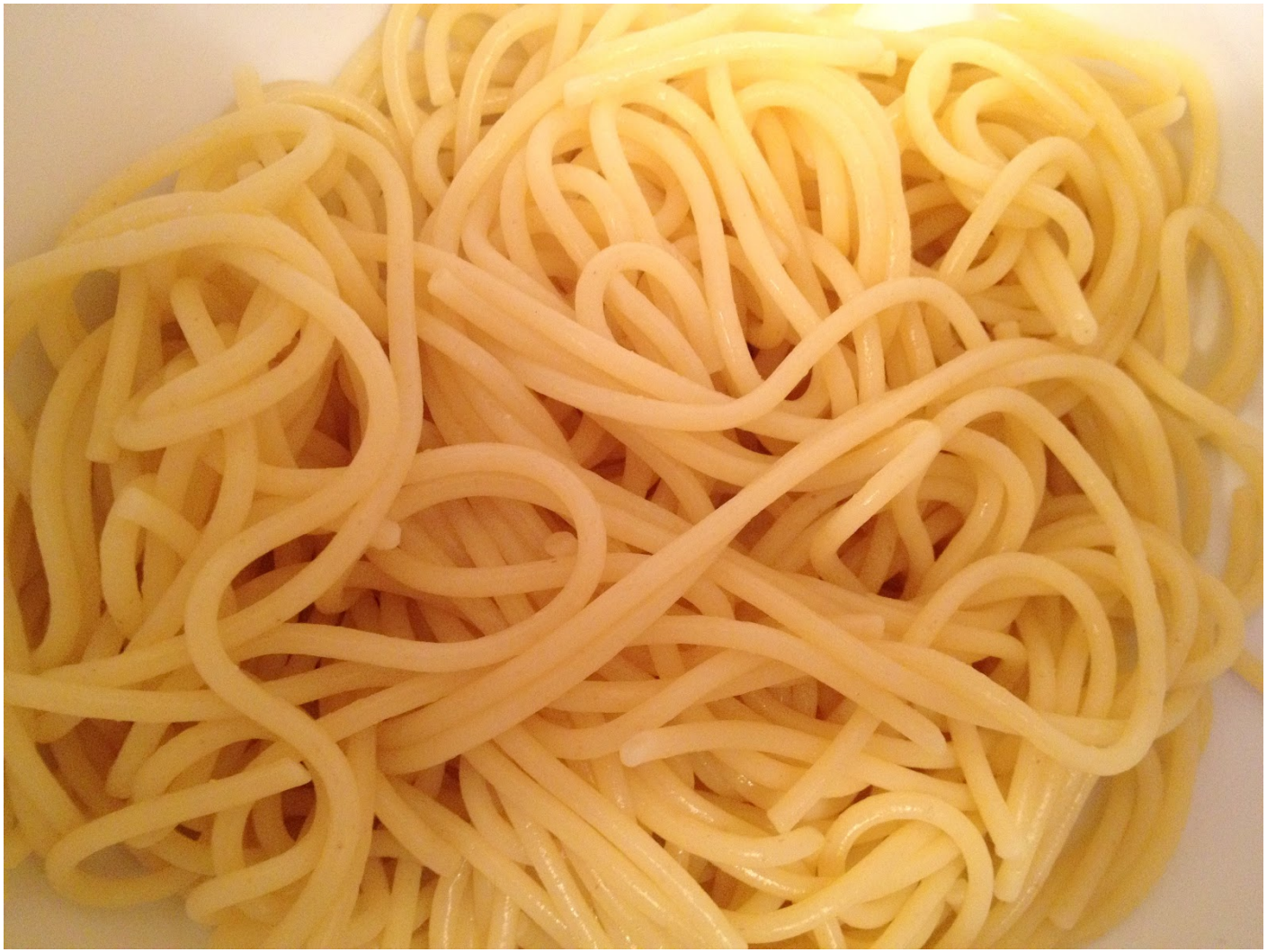


A tangle

What is this ?

What is this ?

A tangle

What is this ?

What is this ?

A link

What is this ?

What is this ?

A braid

What is this ?

What is this ?

A knot

What is this ?

What is this ?

A link made of three knots

What is this ?

What is this ?

A tangle

What is this ?

What is this ?

A link

What is this ?

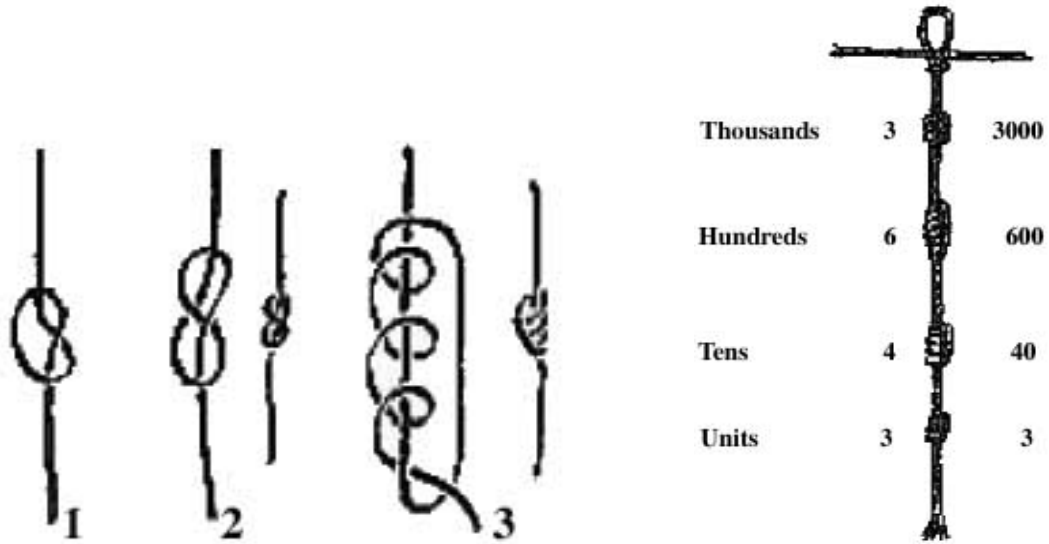
What is this ?

A knot

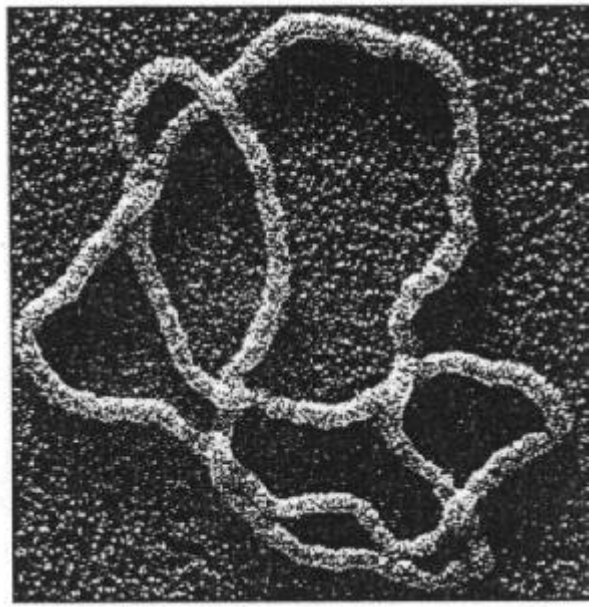
What is this ?

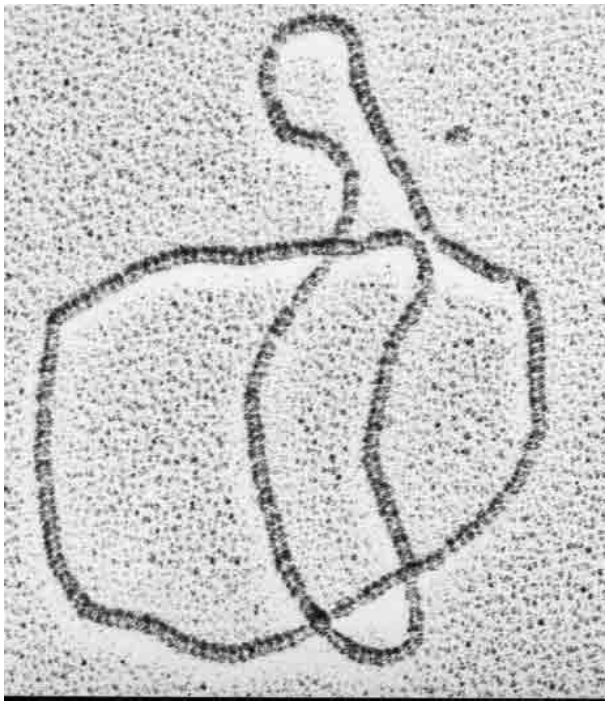
What is this ?

A braid



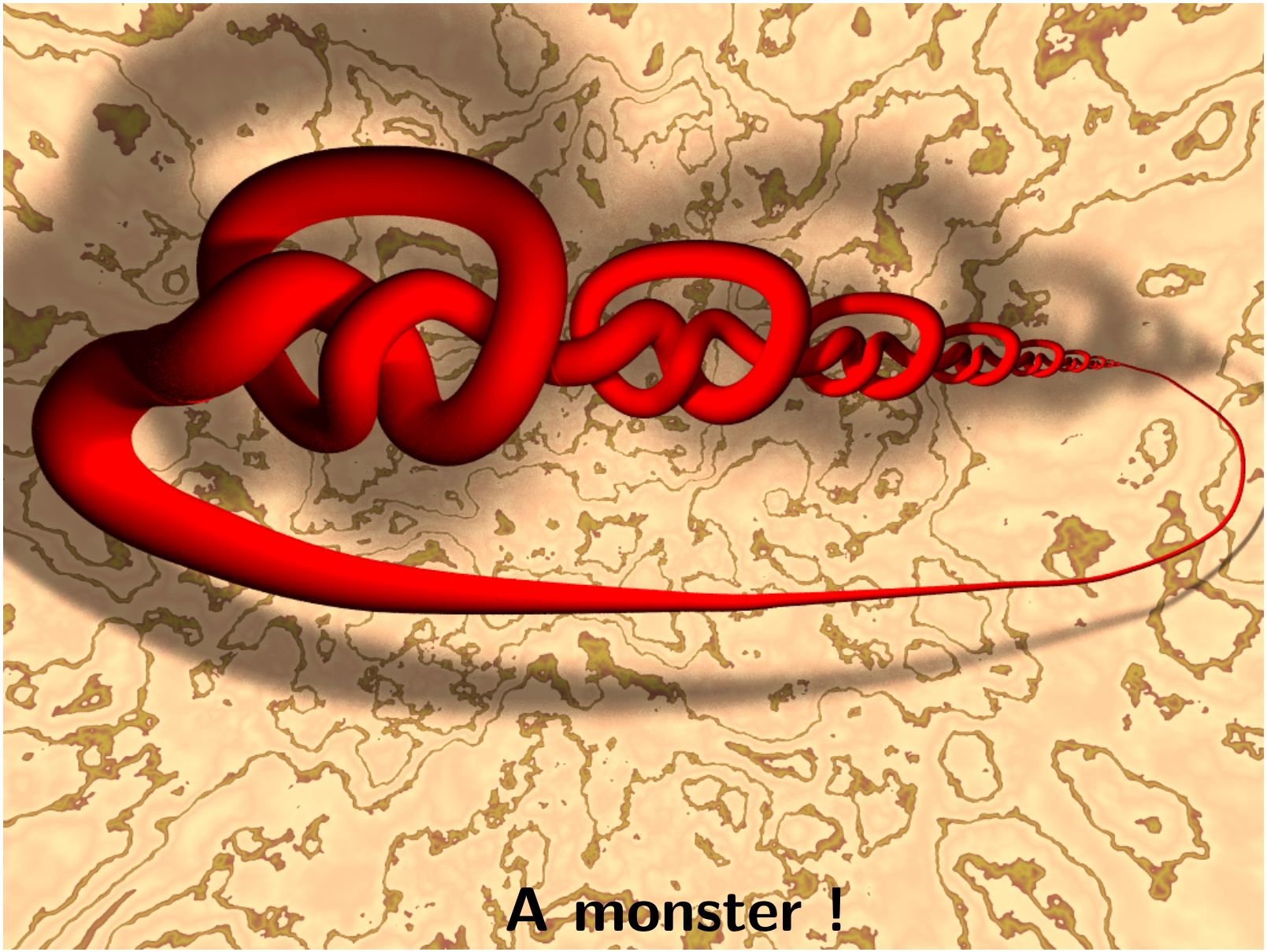
Quipu knots (in fact tangles)



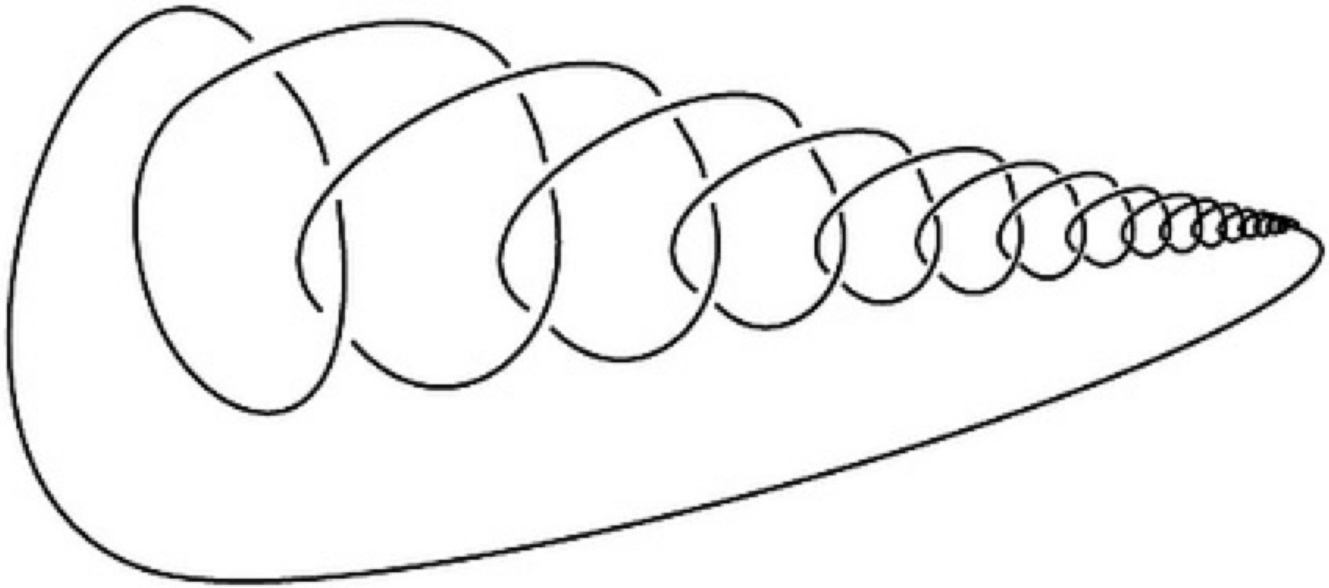


Knotted DNA

Definitions



A monster !

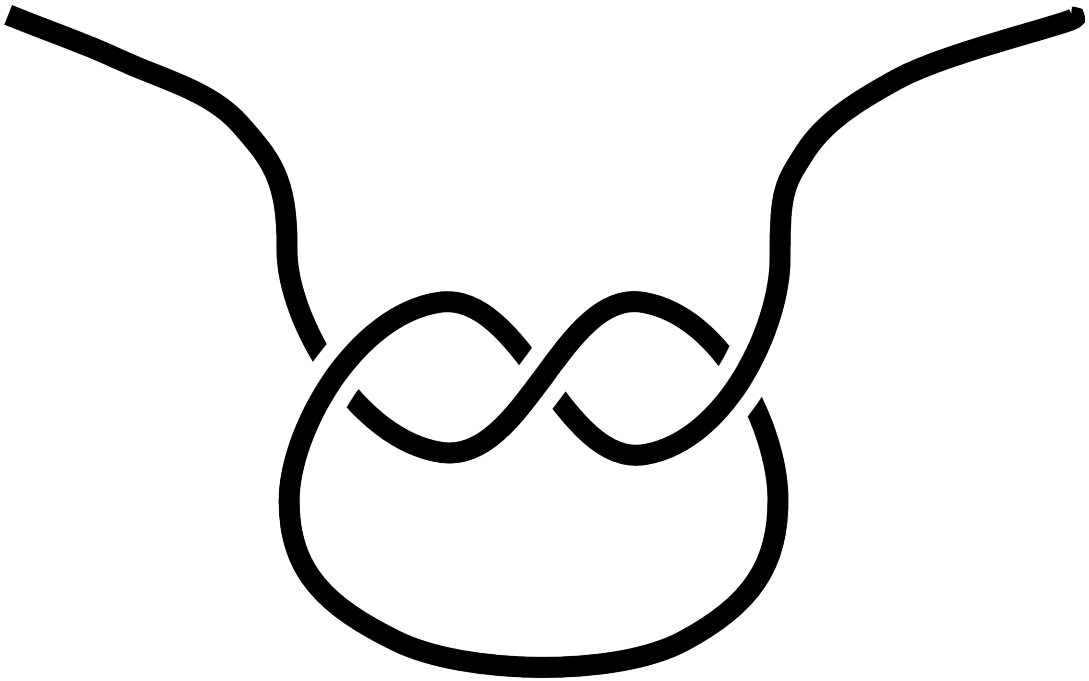


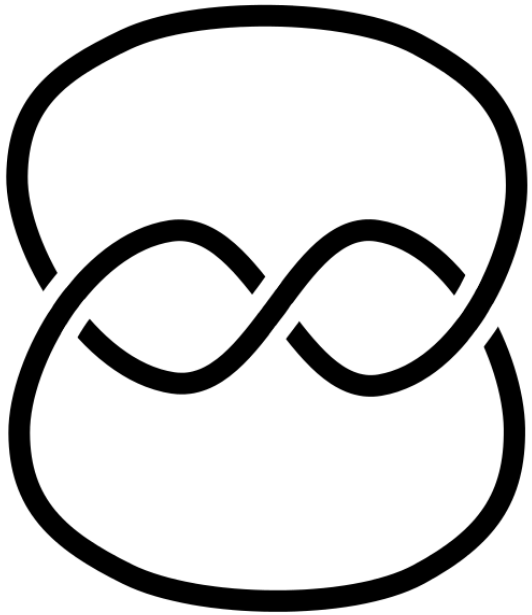
A monster !

We usually just say a knot when we mean an equivalence class of polygonal knots up to ambient isotopy.

Problems

Problems





From shoe lace to knot

Problems

Can we knot with one hand ?

Problems

Can we knot with one hand ?
... and cancel the knot ?

Back to Borromean rings

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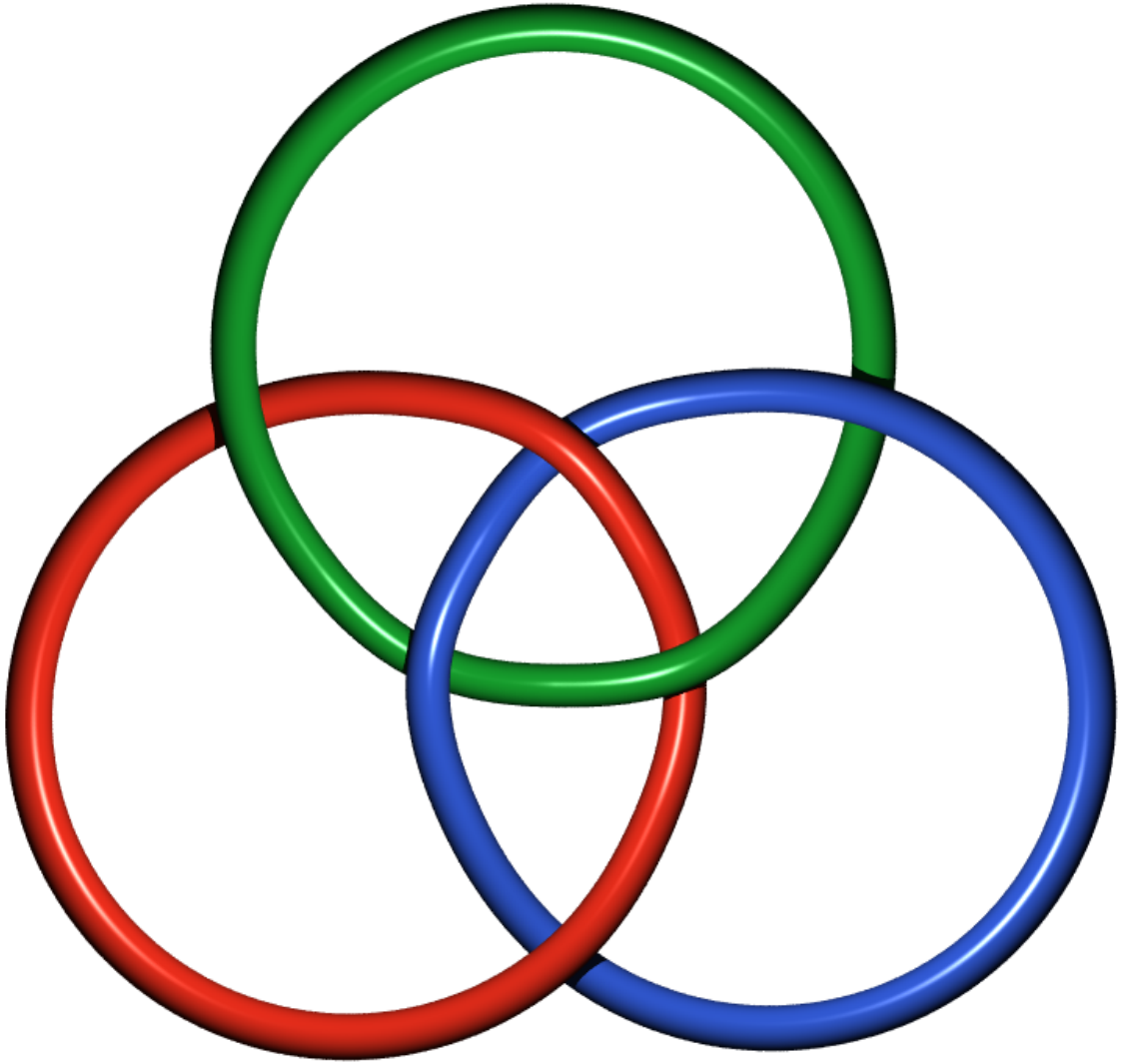


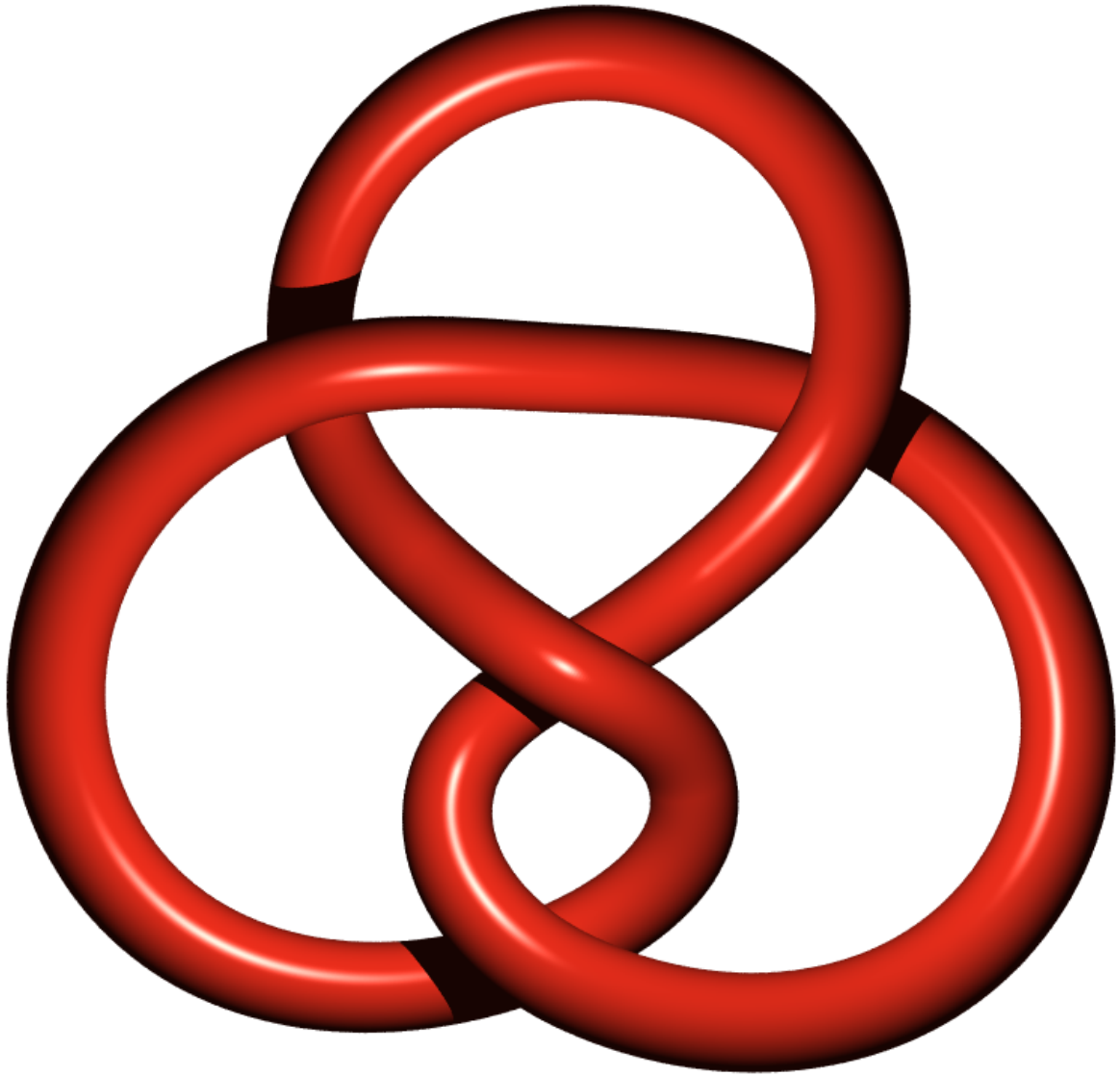


Figure eight knot

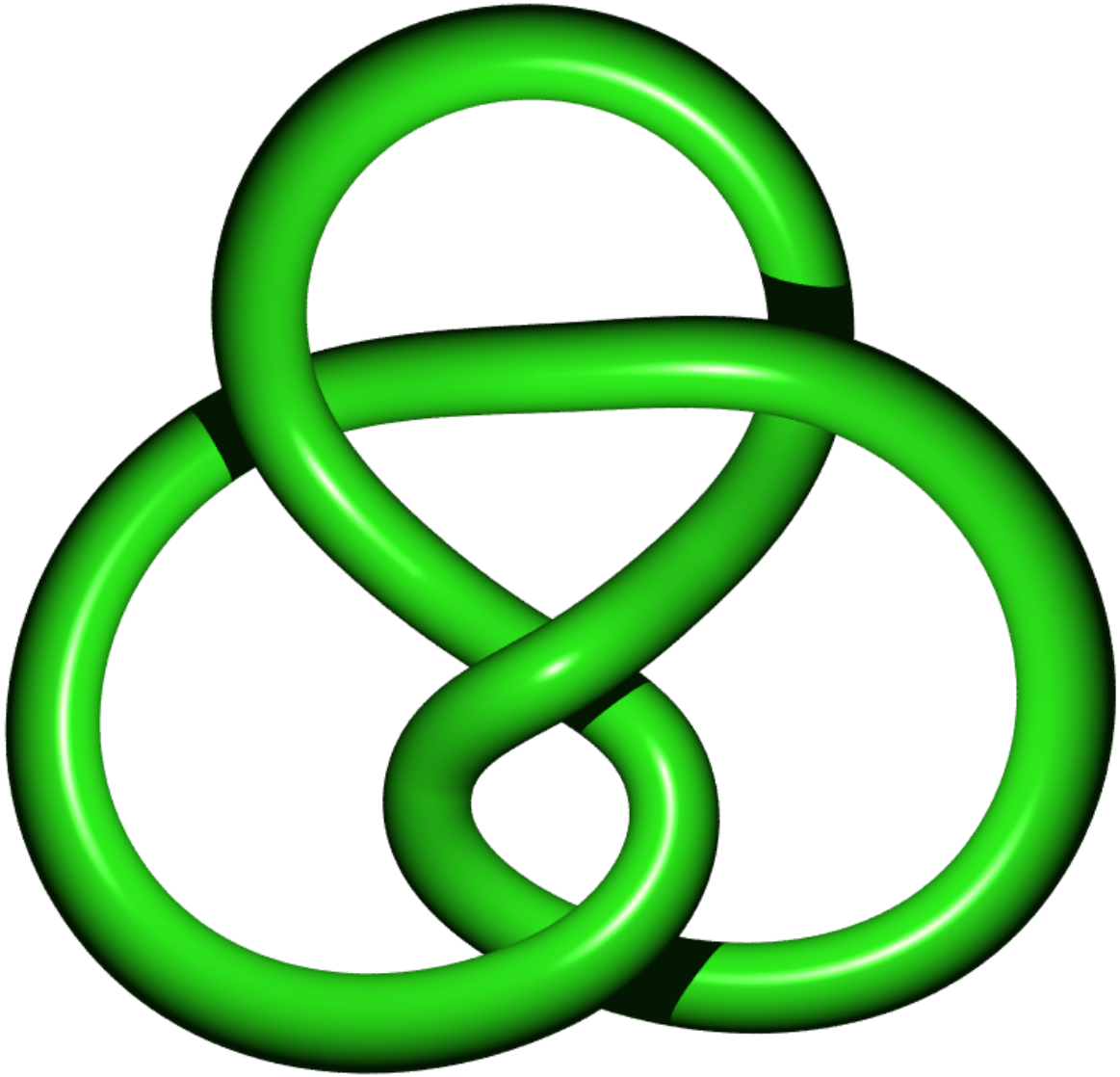
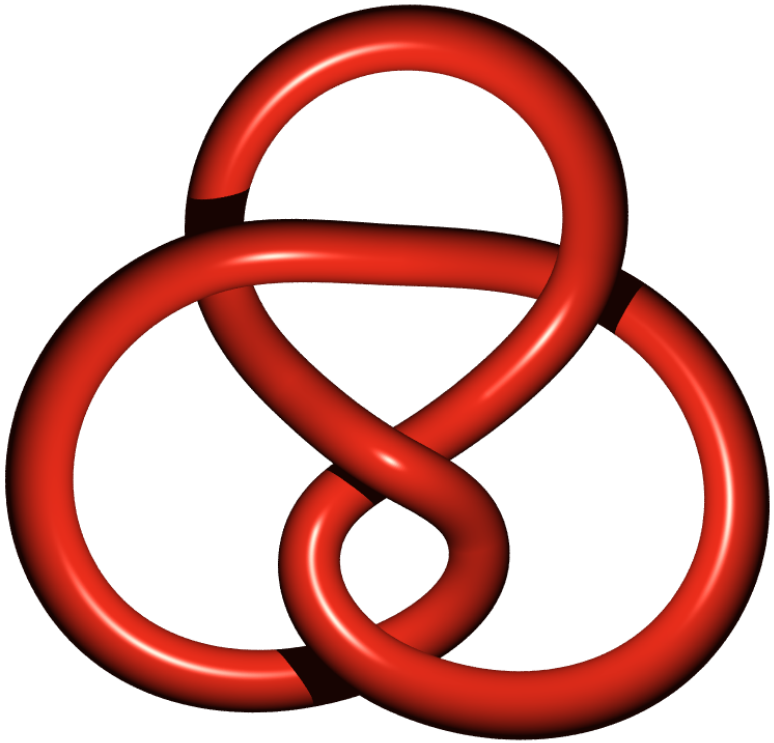
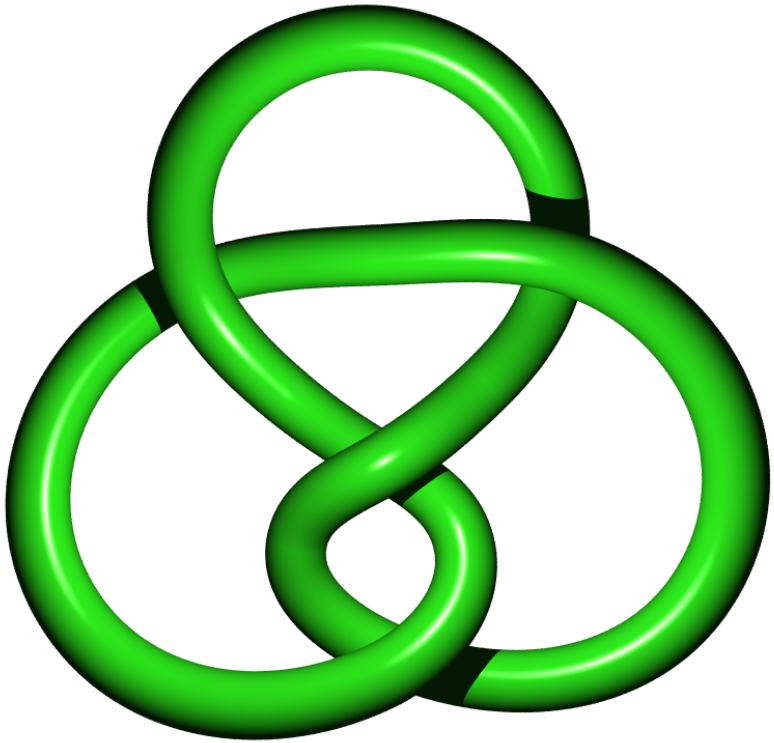


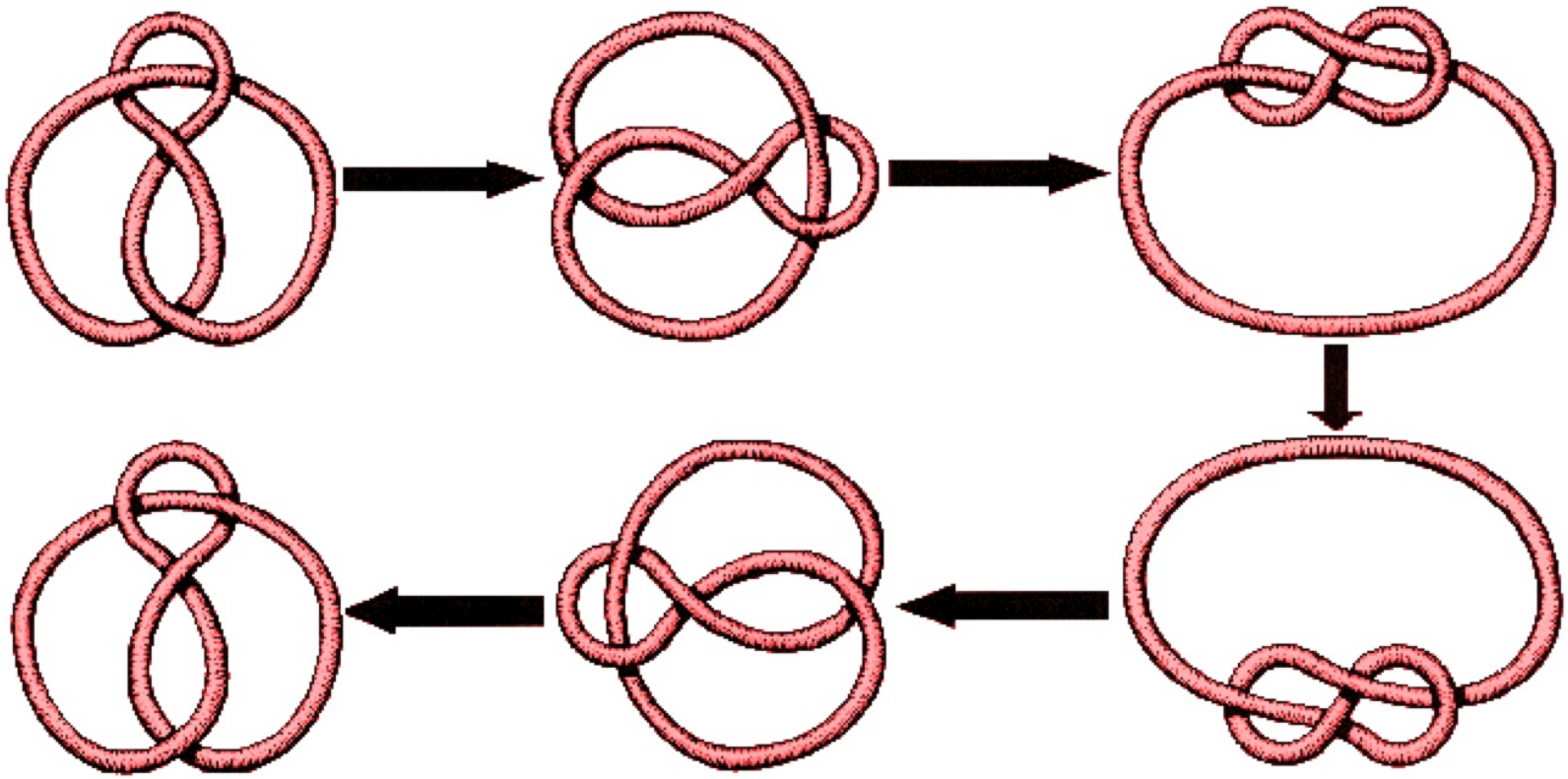
Figure eight knot (mirror)





$=$

Figure eight knot is amphicheiral



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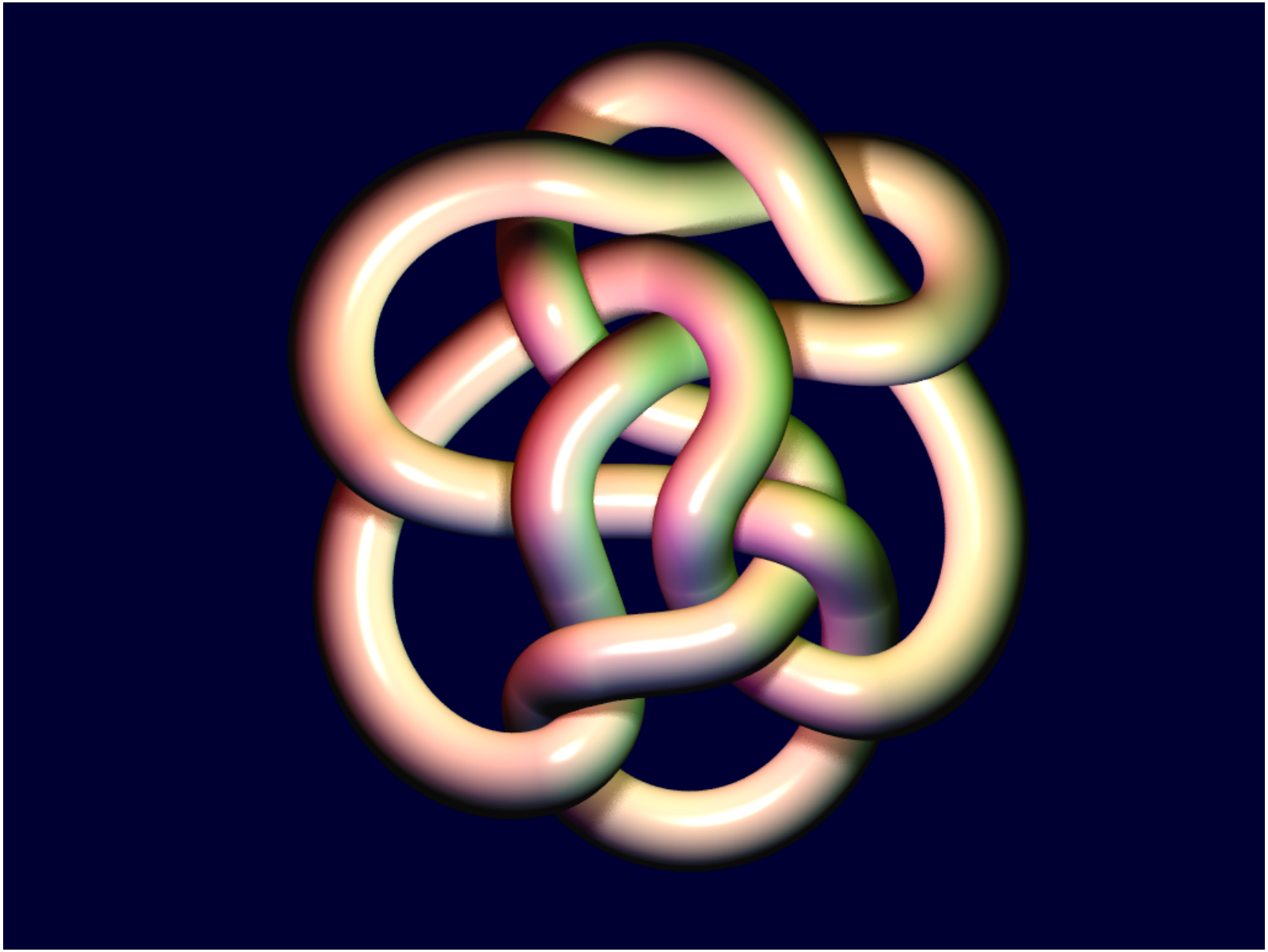

Unknot ?

Diagrams

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A **diagram of a knot** (or any tangle) is a generic projection together with information giving at each crossing which strand goes over the other one.

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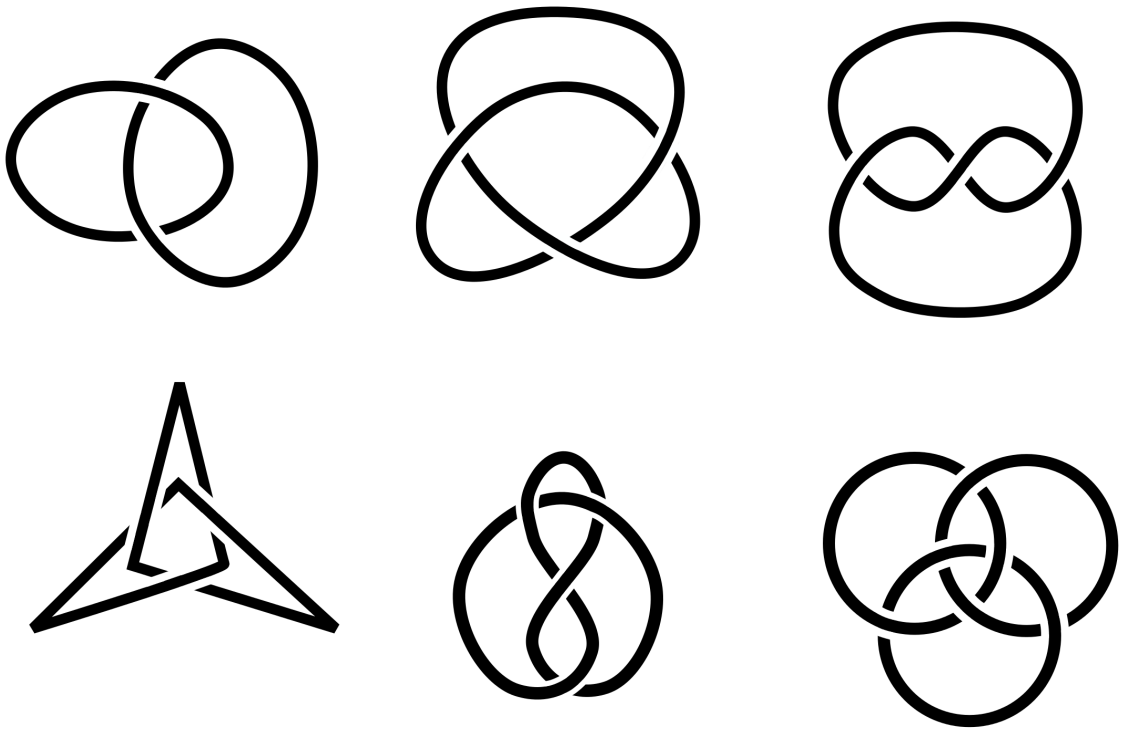
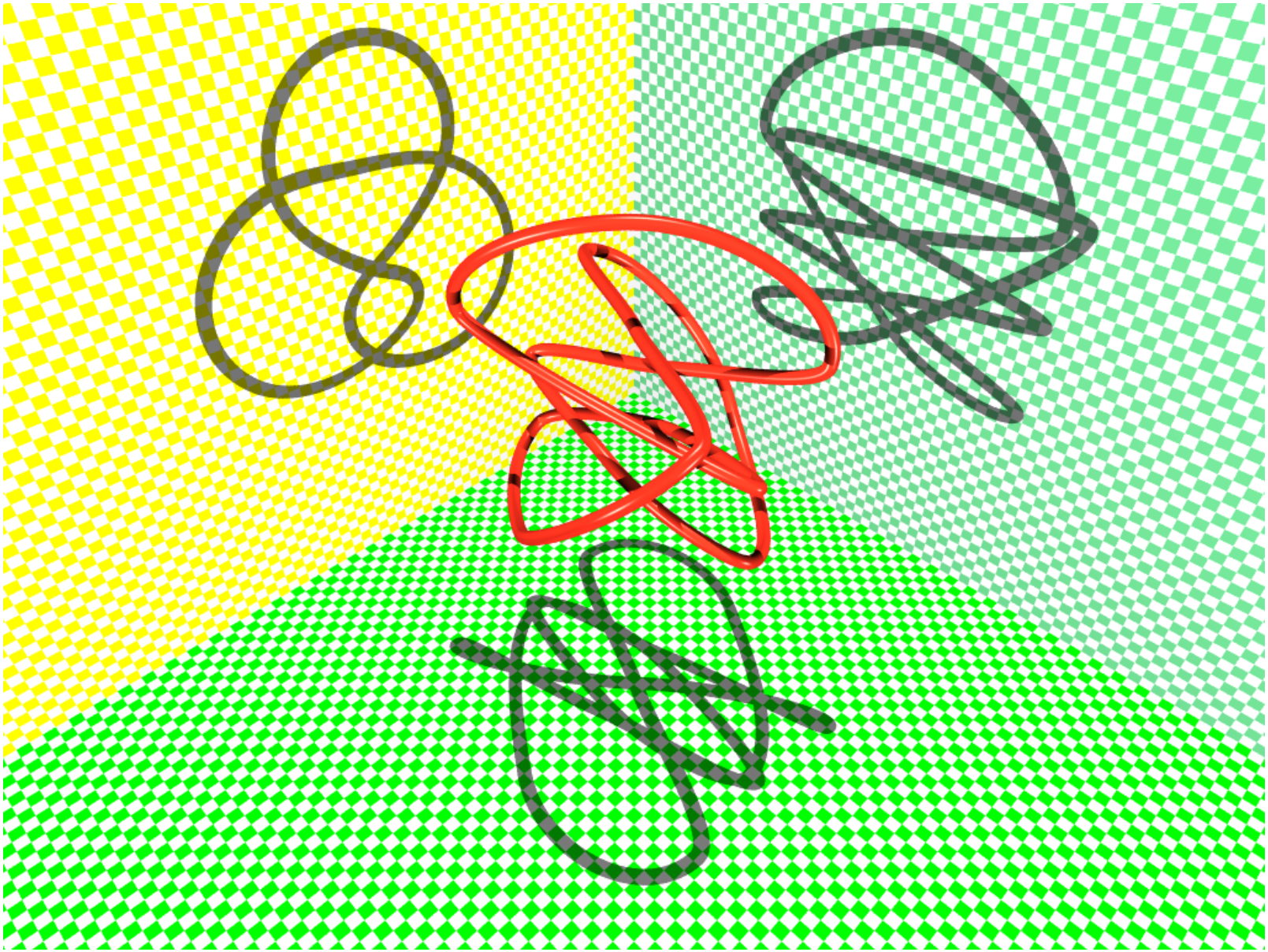
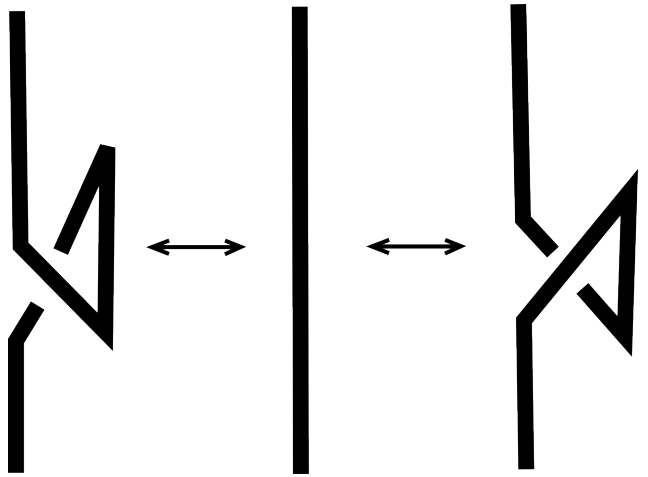
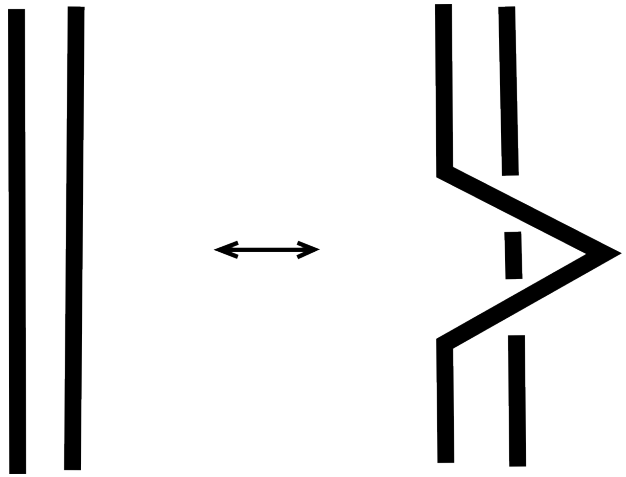




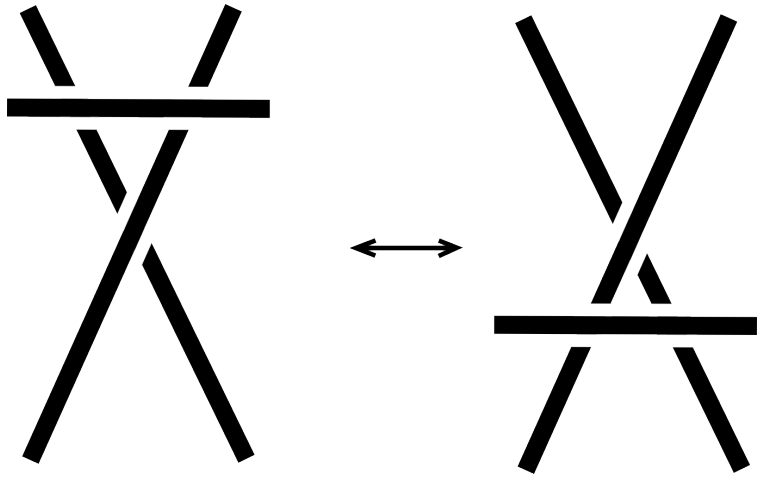
Three shadows of figure eight knot.



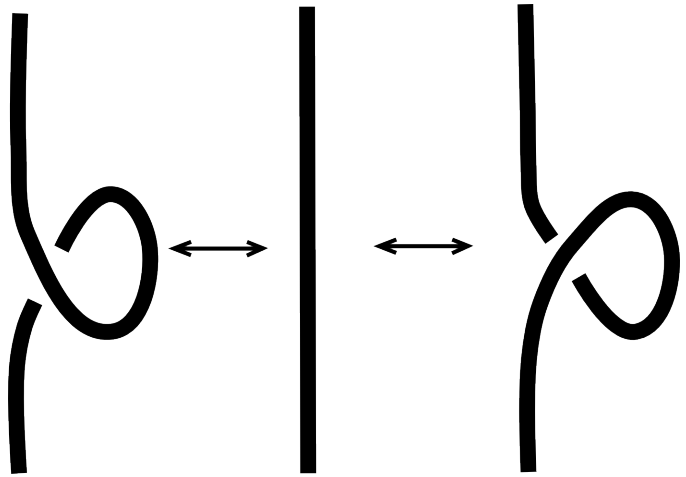
R_1



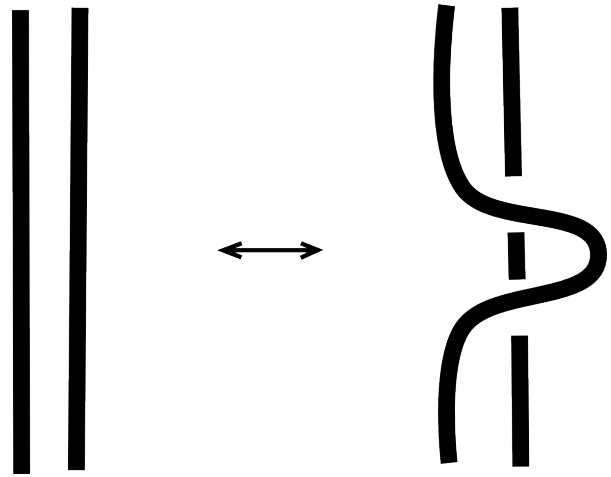
R_2



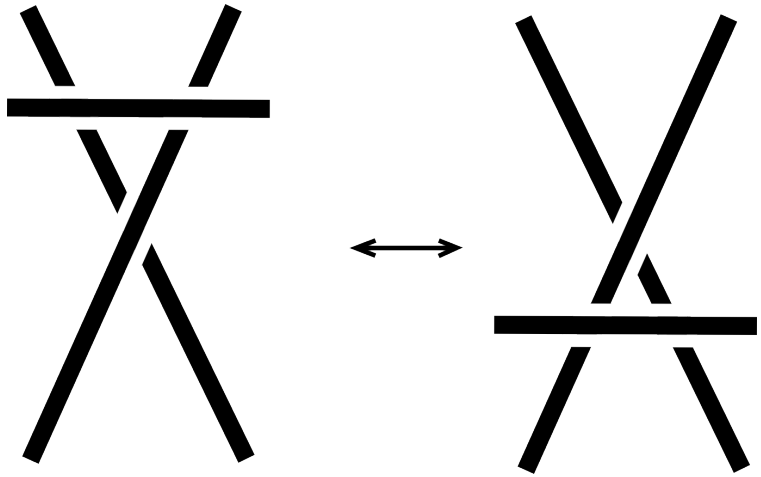
R_3



R_1



R_2



R_3