Analysis IV
Exercise 1
2004

1. Let $\{x_1, \ldots, x_n\}$ be a linearly independent set of vectors in the space $\mathbb{R}^n$. Prove that every $x \in \mathbb{R}^n$ has at most one representation of the form $x = \lambda_1 x_1 + \cdots + \lambda_n x_n$.

2. Prove that $\{e_1, \ldots, e_n\}$ is not a strict subset of any linearly independent set of $\mathbb{R}^n$.

3. Show that the set $F(S, V)$ in Definition 1.5 equipped with addition and scalar multiplication is a vector space.

4. Let U be a linear subspace of V, $x_1, \ldots, x_p \in U$ and $a_1, \ldots, a_p \in \mathbb{F}$. Then $a_1 x_1 + \cdots + a_p x_p \in U$.

5. Let V be a vector space and let U_1 and U_2 be its linear subspaces. Prove that $U_1 \cup U_2$ is a linear subspace of V if and only if $U_1 \subset U_2$ or $U_2 \subset U_1$.

6. Prove: If V is a vector space and $\{U_i\}_{i \in I}$ is a set of linear subspaces of V, then $U = \bigcap_{i \in I} U_i$ is a linear subspace of V.