Analysis IV
Exercise 6
2004

1. Prove that the preimages $f^{-1}(\{r\})$, $r \in \mathbb{R}$, and f^{-1} (any interval) for a measurable function f are measurable.

2. Prove Theorem 2.26 (b').

3. Let $f,\, g\colon E \to \widehat{\mathbb{R}}$ be measurable functions. Prove that the sets

$$\begin{array}{ll} \text{(i)} & \{\, x \in E \mid f(x) < g(x) \,\} \\ \text{(ii)} & \{\, x \in E \mid f(x) \leq g(x) \,\} \end{array}$$

   and

$$\text{(iii)} \quad \{x \in E \mid f(x) = g(x)\}$$

   are measurable (Compare the proof of Theorem 2.17).

4. Let $f_1, \ldots, f_n : E \to \widehat{\mathbb{R}}$ be measurable functions. Prove that the functions

$$\max \{f_1, \ldots, f_n\} \qquad \text{and} \qquad \min \{f_1, \ldots, f_n\}$$

   are measurable.

5. Let f be a nonnegative measurable function. Show that $\displaystyle\int f\, dm = 0$ implies $f = 0$ a.e.