Analysis IV
Exercise 7
2004

1. Let $f : \mathbb{R} \longrightarrow \widehat{\mathbb{R}}$ be a measurable function. If $E \in \mathcal{M}$ and $m(E) = 0$, then $\int\limits_E f dm = 0$.

2. Let $\alpha, \beta \in \mathbb{R}$ and $f, g \in L^\infty$. Prove that

$$\inf\{b |\, |\alpha f + \beta g| \leq b, a.e.\} \leq |\alpha| \inf\{b_1 |\, |f| \leq b_1,\ a.e.\} + |\beta| \inf\{b_2 |\, |g| \leq b_2,\ a.e.\}\,.$$

3. Prove that $d_{L^\infty} : L^\infty \times L^\infty \longrightarrow \mathbb{R}$ is a metric.

4. Prove Lemma 3.1.

5. Prove: If $|f(x)| \leq M$ for $x \in E \in \mathcal{M}$ and $\int\limits_E f dm$ exists, then

$$\Big| \int\limits_E f dm \Big| \leq M m(E)\,.$$