Analysis IV
Exercise 8
2004

1. Prove that if $f \in L^\infty$, $g \in L^\infty$, then $fg \in L^\infty$.
2. Prove Lemma 3.9.
3. Show that if $f \in L^1$ and $g \in L^\infty$, then
$$\int |fg| dm \leq d_{L^1}(f, 0) d_{L^\infty}(g, 0).$$
4. Prove: If the sequence of measurable functions $\{f_n\}$ converges to a measurable function f in the measure m, then $\{f_n\}$ is a Cauchy sequence in the measure m.
5. Prove Lemma 3.15 for $p = \infty$.