Analysis IV
Exercise 11
2004

1. Let c and c_0 be the spaces introduced in Exercise 10, Problem 6. Let

$$c_{0,0} = \Big\{ \{x_n\}_{n=1}^{\infty} : x_n \in \mathbb{R},\ \exists N \in \mathbb{N} \text{ s.t. } x_n = 0, \forall n > N \Big\}.$$

Prove that $c_{0,0} \subset l^p \subset l^\infty,\ c_{0,0} \subset c_0 \subset c \subset l^\infty$.

2. Let $C'([0,1]) = \{f : [0,1] \to \mathbb{R} \,|\, f' \text{ is continuous}\}$. Let

$$\mathcal{B} = \Big\{ f \in C'([0,1]) \,\Big|\, \|f\|_{\mathcal{B}} = |f(0)| + \sup_{x\in[0,1]} (1-|x|^2)|f'(x)| < \infty \Big\}.$$

Prove that $\mathcal{B}$ is a normed space.

3. Let X be the normed space. Prove that the set

$$T = \{x \in X : \|x\| \leq 1\}$$

is closed under the topology induced by the norm.

4. Let X be the normed space and let Y be its open subspace. Prove that $X = Y$.

5. The example on the sheet, page 58, of an overhead projector.