Analysis IV
Exercise 13
10. 5. 2004

1. If $T : C_{\mathbb{R}}[0,1] \to \mathbb{R}$ is the linear transformation defined by
$$T(f) = \int_0^1 f(x)dx$$
show that T is continuous.

2. Let $h \in L^\infty[0,1]$.
   (a) If f is in $L^2[0,1]$, show that $fh \in L^2[0,1]$.
   (b) Let $T : L^2[0,1] \to L^2[0,1]$ be the linear transformation defined by $T(f) = hf$. Show that T is continuous.

3. Let $\mathcal{H}$ be a complex Hilbert space and let $y \in \mathcal{H}$. Show that the linear transformation $f : \mathcal{H} \to \mathbb{C}$ defined by
$$f(x) =< x, y >$$
is continuous.

4. (a) If $(x_1, x_2, x_3, x_4, \cdots) \in l^2$, show that
$$(0, 4x_1, x_2, 4x_3, x_4, \cdots) \in l^2.$$
   (b) Let $T : l^2 \to l^2$ be the linear transformation defined by
$$T(x_1, x_2, x_3, x_4, \cdots) = (0, 4x_1, x_2, 4x_3, x_4, \cdots).$$
   Show that T is continuous.