
Analysis IV

Exercise 9

2004

1. Proof Lemma 3.16 for $p = \infty$.

2. Let $f_n : \mathbb{R} \rightarrow \mathbb{R}$,

$$f_n(x) = \begin{cases} \frac{1}{n^2}, & x \in [-n, n], \\ 0, & \text{elsewhere.} \end{cases}$$

Does $\{f_n\}$ converge

a) pointwise,

b) in the measure m ,

c) with respect to d_{L^p} -metric when $1 \leq p \leq \infty$?

3. Prove Example (*).

4. Let $f_n : [0, 1] \rightarrow \mathbb{R}$, $n \in \mathbb{N}$, be defined by $f_n(x) = x^n$. Find the norm of f_n in the following cases:

(a) in the normed space $C_{\mathbb{R}}([0, 1])$,

(b) in the normed space $L^1([0, 1])$.