Complex analysis
Demonstration 1
28. 9. 2004

For the first three questions, let $\alpha = 3 + 2i, \beta = 1 - 4i, \gamma = \frac{1}{2} + 3i$.

1. Find

   (a) Re α $\qquad$ (b) Re $(\alpha + \beta)$

   (c) Im $(\alpha - \beta)$ $\qquad$ (d) Im $(\alpha - \gamma + \beta)$.

2. Compute $(\alpha\beta)\gamma$ and $\alpha(\beta\gamma)$.

3. Find (a) $1/\alpha$, (b) β/α.

4. If $z = x + iy$ with x and y real, find the following in terms of x and y:

   (a) Re z^2
   (b) Im z^2
   (c) Re $(1/z^2)$
   (d) Im $(1/z^2)$.

5. Show that $\alpha\beta = 0$ $(\alpha, \beta \in \mathbb{C})$ implies at least one of α and β is 0.

6. Show for $\alpha, \beta \in \mathbb{C}$,

   (a) $\alpha + \bar{\alpha} = 2\text{Re } \alpha$ $\qquad$ (b) $\overline{\alpha + \beta} = \bar{\alpha} + \bar{\beta}$

   (c) $(\overline{\alpha/\beta}) = \bar{\alpha}/\bar{\beta}$ $\qquad$ (d) $|\alpha| = |\bar{\alpha}|$.

7. If $P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n$, show

$$\overline{P(z)} = \bar{a}_0 + \bar{a}_1 \bar{z} + \bar{a}_2 \bar{z}^2 + \cdots + \bar{a}_n \bar{z}^n.$$