Complex analysis
Demonstration 10
30. 11. 2004

1. Find the Taylor series of the following functions about the indicated centers:

$$(a) \quad \cos z, \ \frac{\pi}{2} \qquad (b) \quad \cos^2 z, \quad 0.$$

2. Find the Laurent series of the following functions about the indicated annuli:

$$(a) \quad \frac{1}{z(1-z)}, \quad 0 < |z-1| < 1 \qquad (b) \quad \frac{1}{z(1-z)}, \quad |z-1| > 1.$$

3. Find the Laurent series of the following and discuss the character of the function at the center:

$$\frac{1-\cos z}{z^2}, \quad |z| > 0.$$

4. Show $\displaystyle\int_0^{2\pi} e^{2\cos\theta} d\theta = 2\pi \sum_{n=0}^{\infty} \frac{1}{(n!)^2}$.

5. Use the identity theorem to prove

   (a) $\cos^2 z + \sin^2 z = 1$,

   (b) $\sin(z+\alpha) = \sin z \cos\alpha + \cos z \sin\alpha$, where $\alpha \in \mathbb{R}$.