Complex analysis
Demonstration 11
7. 12. 2004

1. Prove that a Möbius transformation $w = \alpha z + \beta$, $\alpha \neq 0$, maps the extended z-plane onto the extended w-plane in a one-to-one manner with $z = \infty$ mapping onto $w = \infty$.

2. Show that $w(z) = \dfrac{\alpha z + \beta}{\gamma z + \delta}$ reduces to $w =$ constant if $\alpha\delta - \beta\gamma = 0$.

3. Show that $w = 1/z$, with the conventions $1/0 = \infty$ and $1/\infty = 0$, maps the extended z-plane in a one-to-one manner onto the extended $w-$plane.

4. Prove Theorem 4.2 a).

5. Show that any two circles from (X) and (Y) which intersect, do so at right angles.

6. If $T(z) = \dfrac{\alpha_1 z + \beta_1}{\gamma_1 z + \delta_1}$ and $S(z) = \dfrac{\alpha_2 z + \beta_2}{\gamma_2 z + \delta_2}$, then show that $T \circ S(z) = T(S(z))$ is in the form
$$T \circ S(z) = \frac{\alpha z + \beta}{\gamma z + \delta}\,.$$