Complex analysis
Demonstration 2
5. 10. 2004

1. In Theorem 1.2 b) prove the equality claim.

2. Prove Corollary 1.2 c).

3. Compute all values of

$$(a)\ (1+i)^{1/2},\quad (b)\ (-16)^{6/8},\quad (c)\ (1+i)^{5/3}.$$

4. Find the limit of the following sequences:

$$(a)\ \frac{1}{n}+\left(\frac{n+2}{n}\right)i,\quad (b)\ \left(\frac{1}{\sqrt{3}}+\frac{i}{\sqrt{3}}\right)^n,\quad (c)\ \left(1+\frac{1}{n}\right)^n+\left(1+\frac{1}{n}\right)^{-n}i.$$

5. If $\lim z_n=\alpha$ and $\lim \zeta_n=\beta$, then show

$$(a)\ \lim(z_n\pm\zeta_n)=\alpha\pm\beta,\quad (b)\ \lim(z_n\zeta_n)=\alpha\beta.$$

6. For $|z|<1$ show that

$$\lim nz^n=\lim n^2z^n=\lim n(n+1)z^n=0.$$

7. Compute the following integrals

$$(a)\ \int_0^1(2+ip^2)dp\,,\quad (b)\ \int_{-2}^5 f(s)ds,\quad \text{where}\quad f(s)=\begin{cases}-1+7is^2, & -2\le s<2,\\ 2s+is^2, & 2\le s\le 5.\end{cases}$$

8. Use Theorem 1.4 e) to show

$$\left|\int_C(3+2y)dz\right|\le 54\pi,$$

where C is the circle $|z|=3$.