
Complex analysis

Demonstration 3

12. 10. 2004

1. Let C be the circle $|z| = 1$. Show

$$\left| \int_C \frac{dz}{2z^2 + 5} \right| \leq \frac{2\pi}{3}.$$

2. Evaluate:

$$\int_C \frac{dz}{z^2 - 4}, \text{ when } C \text{ is (a) } |z| = 1; \text{ (b) } |z| = 4; \text{ (c) } |z - 2| = 2.$$

3. Evaluate:

$$\int_C \frac{dz}{z^2 - 1}, \text{ when } C \text{ is (a) } |z| = \frac{1}{3}; \text{ (b) } |z| = 3; \text{ (c) } |z - 1| = 1.$$

4. Evaluate:

$$\int_C \frac{dz}{z(z-1)(z+2)}, \text{ when } C \text{ is (a) } |z| = \frac{1}{4}; \text{ (b) } |z| = \frac{5}{4}; \text{ (c) } |z| = 3; \text{ (d) } |z-1| = \frac{1}{2}.$$

5. Evaluate:

$$\int_0^{2\pi} \frac{d\theta}{3 \sin \theta + 5}.$$

6. Evaluate:

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}.$$

7. Show that

$$\int_0^{2\pi} \frac{d\theta}{1 + a \cos \theta} = \frac{2\pi}{\sqrt{1 - a^2}}, \quad 0 < a < 1.$$