1. Compute by definition the derivatives of

$$(a) \quad z^3 + z^2 - z + 1 \qquad (b) \quad \frac{z^2 - 1}{z^2 + 1} \qquad (c) \quad (z^2 - 1)(z^2 - 3z).$$

2. Show that if $f'(\alpha)$ exists, then $f(z)$ is continuous at $z = \alpha$.

3. Show that each polynomial $P(z)$ is analytic everywhere.

4. Show that the exponential function $f(z) = e^z$ is an entire function (one can suppose that f is analytic at the origin).

5. Show that if $f(z) = u(z) + iv(z)$ is analytic in a region $\mathcal{D}$, and either (a) u, (b) v, (c) $|f(z)|$ is a constant, then $f(z)$ itself must be constant.

6. Show that the following functions are nowhere differentiable:

$$(a) \quad f(z) = \bar{z} \qquad (b) \quad f(z) = x.$$