Complex analysis
Demonstration 5
26. 10. 2004

1. If $f(z) = u + iv$ is entire (kokonainen), and if $u = x^3 - 3xy^2$, find v from the Cauchy-Riemann equations, and express $f(z)$ as a polynomial in z, which is unique up to a pure imaginary constant.

2. Show that $f(z) = (1+z^2)/(z^2-1)$ is analytic at ∞.

3. Test $\sum \alpha_n$ for convergence where α_n is

$$(a)\ \frac{n!}{n^n} \qquad (b)\ \frac{n^3(n+1)^n}{(3n)^n} \quad (\text{use} \quad \sqrt[n]{n} \to 1 \quad \text{as} \quad n \to \infty).$$

4. Find the radius of convergence of $\sum \frac{z^k}{k^2}$.

5. Find the radius of convergence of $\sum \left(\frac{n+1}{n}\right)^{n^2} z^n$.

6. Find the radius of convergence of $\sum \frac{1}{n^p} z^n$ and $\sum n^p z^n$, $p > 0$.