Complex analysis
Demonstration 8
16. 11. 2004

1. let C be a contour with $\alpha \in I(C)$.

   (a) Evaluate $\int\limits_C \frac{e^z}{z-\alpha} dz$.

   Hint: By the definition of the exponential

   $$\frac{e^z}{z-\alpha} = \frac{e^\alpha e^{z-\alpha}}{z-\alpha} = \frac{e^\alpha}{z-\alpha} + e^\alpha \sum_1^\infty \frac{(z-\alpha)^{n-1}}{n!} = \frac{e^\alpha}{z-\alpha} + g(z),$$

   where $g(z)$ is entire.

   (b) Evaluate $\int\limits_C \frac{e^z}{(z-\alpha)^n} dz$, $n \geq 2$.

2. Integrate

   $$\int\limits_\gamma \frac{\cos z}{z^3+z} dz$$

   over the given curves: (a) $\gamma : |z| = 2$, (b) $\gamma : |z| = \frac{1}{2}$, and (c) $\gamma : \left|z - \frac{i}{2}\right| = 1$.

3. Let $f(z)$ be analytic and bounded by M in $|z| \leq R$. Prove that

   $$|f^{(n)}(z)| \leq \frac{MRn!}{(R-|z|)^{n+1}}, \quad |z| < R.$$

4. Evaluate $\int\limits_\gamma e^z dz$, where γ is the semicircle from -1 to 1 passing through i.

5. Specialize Theorem 3.2 a) to the case where z is the center of the circle and show that

   $$f(z) = \frac{1}{2\pi} \int\limits_0^{2\pi} f(z + re^{it}) dt.$$

6. Evaluate

   $$\int\limits_C \frac{\sin z}{z-\alpha} dz,$$

   where C is a contour with $\alpha \in I(C)$ (Hint: Use the technique of exercise 1).