Complex analysis
Demonstration 9
23. 11. 2004

1. Suppose that $f(z) = u(z) + iv(z)$ is an entire function and there exists $M > 0$ such that $u(z) \leq M$. Show that $f(z)$ is constant.

2. Find the Taylor series of the following functions about the indicated centers

$$(a)\ \ \frac{1}{z},\quad 1\ ;\qquad (b)\ \ \frac{1}{z},\quad 2\ ;\qquad (c)\ \ \mathrm{Log}(1+z),\quad 0.$$

3. Find the Taylor series of $\sin z$ about $\pi/2$.

4. Find the Laurent series of the following functions in the indicated annuli:

$$(a)\ \ \frac{1}{1-z},\quad |z| > 1\ ;\qquad (b)\ \ \frac{\sin z}{z^3},\quad |z| > 0.$$

5. Suppose $f(z)$ is analytic for $|z| < 1$ and $|f(z)| \leq M$. If $f(z) = \sum_0^\infty a_n z^n$ show that $\sum_0^\infty |a_n|^2 \leq M^2$. Hint: For $r < 1$

$$|f(re^{i\theta})|^2 = f(re^{i\theta})\overline{f(re^{i\theta})} = \Big(\sum_0^\infty a_n r^n e^{in\theta}\Big)\Big(\sum_0^\infty \bar{a}_k r^k e^{-ik\theta}\Big),$$

then integrate about θ from 0 to 2π.