Numerical linear algebra autumn 2004

Problem set 1

1. Let us consider the following matrix

$$A = \begin{pmatrix} 2 & -1 \\ 4 & 0 \\ 0 & 1 \end{pmatrix}$$

   What is the image of A ? What is the kernel of A ? What is the rank of A ?

2. Let A be as above. Compute $\|A\|_1$, $\|A\|_\infty$, $\|A\|_F$, $\|A^*\|_1$, $\|A^*\|_\infty$ and $\|A^*\|_F$. Is it always true that $\|A\|_F = \|A^*\|_F$? Is it always true that $\|A\|_1 = \|A^*\|_\infty$?

3. Show that $\|AB\| \leq \|A\| \|B\|$ for p – norms and Frobenius norm.

4. Let us consider the following norm:

$$\|A\| = \max_{i,j} |a_{ij}|$$

   Show that this is a norm. Then show by example that there are matrices A and B such that $\|AB\| > \|A\| \|B\|$ (it is enough to consider 2×2 matrices).

5. Suppose that U is unitary matrix. Show that

   – $|Ux|_2 = |x|_2$ for all x .

   – $\|UA\|_2 = \|A\|_2$ for all A .

   – $\|UA\|_F = \|A\|_F$ for all A .