Numerical linear algebra autumn 2004

Problem set 9

1. Let us consider the following matrices:

$$A = \begin{pmatrix} 2 & -1 \\ 0.5 & -0.1 \end{pmatrix} \qquad \text{and} \qquad B = \begin{pmatrix} 2.4 & 1.2 \\ -2 & 0.8 \end{pmatrix}$$

   Can you use the fixed point iteration to solve systems $Ax = b$ and $Bx = b$? How fast is the convergence? How many iterations are needed to get a reasonable accuracy?

2. Let $A \in \mathbb{C}^{n \times n}$ and suppose that $\operatorname{rank}(A) = k < n$. Show that $\dim(\mathcal{K}_m) \leq k + 1$ for all m.

3. In the GMRES algorithm one has to solve the following least squares problem at each iteration

$$\tilde{H}_m c = |b| \, e^1$$

   where $\tilde{H}_m$ is a $(m + 1) \times m$ Hessenberg matrix. The basic method of doing this is to use QR–factorisation of $\tilde{H}_m$. Try to use this as efficiently as possible. Note that we already know/have computed the QR–factorisation of $\tilde{H}_{m-1}$. You can experiment first with Matlab to see how the factorisations are related. Note also the specific right hand side.

4. The GMRES algorithm as given in the lectures started with the initial guess $x^0 = 0$ and hence with initial residual $r^0 = b$. Now suppose that you have a reasonable initial guess $x^0 \neq 0$. How do you modify the algorithm in this case?

5. Do some experiments with Matlab's command gmres. In particular take some orthogonal matrices and use gmres to them. The convergence is typically very slow. Show with experiments that typically you really need all n iterations to get acceptable accuracy.