Wavelets, spring 2002

Problem set 2

Let us define *Heaviside* function or unit step function:

$$H(t) = \begin{cases} 1\ , & t > 0 \\ 0\ , & t < 0 \end{cases}$$

1. Check that $||\cdot||_1$ is a norm in $L^1(\mathbb{R})$.

2. Let $f(t) = H(t) - H(t-2)$ and $\psi(t) = te^{-|t|}$. Compute the wavelet transform of f. Analyse the behavior of the transform when the scaling parameter a approaches zero.

3. Are the following functions wavelets?

$$\psi_1(t) = \begin{cases} e^{-t} & t \geq 0 \\ -e^{t} & t < 0 \end{cases}$$

$$\psi_2(t) = \begin{cases} e^{-t}\sin(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$

4. Let ψ be a wavelet. Can its Fourier transform $\hat{\psi}$ be also a wavelet? Produce an example or prove that it is not possible.

5. Let us consider the following function:

$$f(t) = \begin{cases} e^t|t|^{4/3}\ , & t < 0 \\ \sin(\pi t)\ , & 0 \leq t \leq 1 \\ \sqrt{t-1}\ , & 1 < t \leq 2 \\ e^{2-t}\ , & t > 2 \end{cases}$$

   Examine the singular points of this function/signal. What are their Lipschitz numbers?

6. Experiment with *Matlab's Wavelet toolbox*. There is a command cwt which performs continuous wavelet transform. However, it is more convenient to use graphical interface command wavemenu. After launching wavemenu, choose Continuous wavelet 1–D.

- Check first some of the examples in the Example analysis in the File menu.
- Analyse the function/signal given in previous exercise.
- In the web pages of the course you can find some test signals. Download them and analyse them.
- Try your own favorite signals.

Of course *Matlab* doesn't really compute the continuous transform. It computes a numerical approximation to it. However, you can freely choose the scales where the approximation is computed.

If you have problems with *Matlab* you can contact Kirsi Helkala:
kirsi.helkala@joensuu.fi