Wavelets, spring 2002

Problem set 5

1. Let φ be the Haar scaling function. Check directly in this case that the function a as defined in Lemma 6.1 is identically equal to one.

2. Let $P : L^2(\mathbb{R}) \to L^2(\mathbb{R})$ be a linear map. P is called a projection if $P \circ P = P$. A projection is orthogonal if for all $f,\ g \in L^2(\mathbb{R})$

$$\langle Pf, g\rangle = \langle f, Pg\rangle$$

   Let φ be a scaling function and ψ the associated wavelet and define

$$P_j f = \sum_{k=-\infty}^{\infty} \langle f, \varphi_{j,k}\rangle \varphi_{j,k}$$

$$Q_j f = \sum_{k=-\infty}^{\infty} \langle f, \psi_{j,k}\rangle \psi_{j,k}$$

   Show that P_j and Q_j are orthogonal projections.

3. Let $m_0(\omega) = \sum_{k=-\infty}^{\infty} h_k e^{-i\omega k}$ and assume that

$$|m_0(\omega)|^2 + |m_0(\omega + \pi)|^2 = 1 \qquad m_0(0) = 1$$

   Show that

$$\sum_{k=-\infty}^{\infty} h_{2k} = \sum_{k=-\infty}^{\infty} h_{2k+1} = 1/2$$

   Hint: compute $m_0(\omega) \pm m_0(\omega + \pi)$.

4. Compute h such that the associated ψ has at least 2 vanishing moments. Proceed like in the last problem of Problem set 4 and assume initially that m_0 is like in equation (7.1) in the notes. Then check that the property in Theorem 7.2 holds. Also estimate numerically the smoothness of the wavelet with help of Theorem 7.3.