Wavelets, spring 2002

Problem set 6

1. Check that downsampling and upsampling are adjoints of each other, see Definition 8.2.

2. Let us define

$$c^3 = \big(\ldots, 0, 7, 2, -1, 3, -2, 4, 5, 3, 0, \ldots\big)$$

   Do the decomposition as in Theorem 8.1 using Haar wavelets. Then do the reconstruction as in Theorem 8.2, and check that you get back c^3.

   Note that at each stage of the algorithm you have the same amount of information, i.e. 8 real numbers.

3. Use *Matlab* to do the previous problem. First launch wavemenu, and then Wavelet 1-D. Choose Haar as the wavelet, and make sure that the results *Matlab* gives are the same as what you computed.

4. Daubechies wavelets (db1=haar, ..., dbn) are designed to have maximum number of vanishing moments. Choose wavemenu, and then Wavelet Display. Plot some of Daubechies wavelets. As you see when the order grows, the support becomes larger and the wavelets tend to become more regular, as predicted by Theorem 7.3.

   Symlets (sym2,... in *Matlab*) are more symmetric than Daubechies wavelets which may be useful in some situations. Note that it's not possible to have exact symmetry. Coiflets (named after Ronald Coifman, coif1,... in *Matlab*) are wavelets where also scaling function has some vanishing moments. Of course the zeroth moment of the scaling function can't vanish because $\int_{-\infty}^{\infty} \varphi(t)dt = 1$, but it's perfectly possible that

$$\int_{-\infty}^{\infty} t^m \varphi(t)dt = 0 \qquad \text{for some} \quad m \geq 1$$

   This is useful in applications where one wants to compress a smooth signal.

   Then there is the Meyer wavelet (meyr in *Matlab*). It doesn't have a compact support, but it's infinitely differentiable.

   You can find also some other wavelets, and there is some information about them. Note that all of them do not come from MRA, in other words they may be suitable for continuous wavelet transform, but not for discrete (and fast) wavelet transform.

5. Experiment with the given signals in *Matlab* and the signals given for Problem set 2, or use your own data.

   - What is the effect of the choice of the wavelet?
   - How many levels should be used in the decomposition?
   - Note the difference between reconstruction, and the coefficients. You can compare the two when choosing Statistics.
   - Compression can be done with Compress, and denoising with De-noise. These are quite closely related, but De-noise allows you to make some assumptions about the nature of the noise.