

Wavelets, spring 2002

Problem set 7

1. Let us define

$$x = (\dots, 0, 1, -1/2, 1/4, -1/8, 1/16, \dots)$$

Compute the Fourier- and \mathcal{Z} - transform of x .

2. Let $y = h*x$ where h is the Haar filter and x as in the previous problem. What is the \mathcal{Z} - transform of y ?
3. Choose an input to the filter bank of Figure 9.1, and let the filters be the Haar filters. Do all the computations and check that you really get the perfect reconstruction.
4. Check the \mathcal{Z} - transform part of Lemmas 9.1 and 9.2.
5. Let us put

$$M_0(z) = \left(\frac{1+z^{-1}}{2}\right)^2 (t_0 + t_1 z^{-1} + t_2 z^{-2} + t_3 z^{-3})$$

Compute some filters by solving equation (9.4). Recall that one must also impose $M_0(1) = 1$. Plot some scaling functions and wavelets with `skaalaf.m`. Can you still find solutions with 3 vanishing moments? In other words suppose that

$$M_0(z) = \left(\frac{1+z^{-1}}{2}\right)^3 (t_0 + t_1 z^{-1} + t_2 z^{-2})$$

Or 4 vanishing moments:

$$M_0(z) = \left(\frac{1+z^{-1}}{2}\right)^4 (t_0 + t_1 z^{-1})$$