Wavelets, spring 2002

Problem set 8

1. The transfer function of the filter can be written as

$$T(\omega) = \sum_{k=-\infty}^{\infty} t_k e^{-i\omega k} = |T(\omega)| e^{i\varphi(\omega)}$$

   where φ is the phase. A filter is said to be causal if $t_k = 0$ for $k < 0$, and to have zero phase if φ is a zero function. Check that

   - T has a zero phase if $T(\omega) = T(-\omega)$. What does this imply about the coefficients t_k ?
   - a causal filter can have zero phase only if $t_k = 0$ for $k \neq 0$.

2. Choose an input to the filter bank of Figure 9.2, and let the filters be the following (one of the Burt's filters):

$$\begin{aligned}
A(\omega) =& \frac{1}{2} + \frac{1}{4}\big(e^{i\omega} + e^{-i\omega}\big) = \frac{1}{2}\big(1 + \cos(\omega)\big) \\
C(\omega) =& \frac{3}{4} + \frac{1}{4}\big(e^{i\omega} + e^{-i\omega}\big) - \frac{1}{8}\big(e^{2i\omega} + e^{-2i\omega}\big) = \frac{1}{4}\big(3 + 2\cos(\omega) - \cos(2\omega)\big) \\
b_k =& (-1)^k c_{1-k} \\
d_k =& (-1)^k a_{1-k}
\end{aligned}$$

   Do all the computations and check that you really get the perfect reconstruction.

3. Consider again Figure 9.2. Let us put

$$A(z) = \Big(\frac{1+z}{2}\Big)^3$$

   Compute some filters B, C and D which correspond to this choice.