Wavelets, spring 2002

Problem set 9

1. Plot the scaling functions and wavelets that correspond to the filters that you computed in the last exercise of the problem set 8. Use the program `skaalaf`. Plot also the (absolute values of) transfer functions.

2. Let $v^1 = (1,0,0)$, $v^2 = (1,1,0)$ and $v^3 = (1,1,1)$. Check that this is a Riesz basis in $\mathbb{R}^3$. What are the constants in the definition of Riesz basis in this case?

   In fact all bases in $\mathbb{R}^n$ are Riesz bases. In the language of linear algebra this can be formulated as follows. Let V be a matrix whose columns are the basis vectors. Then the constants are σ_1^2 and σ_n^2 where σ_1 and σ_n are the largest and smallest singular values of V. You may check this with *Matlab* in the above case, using the command `svd`.

3. Let h^a and h^s be perfect reconstruction filters and m_0^a and m_0^s the corresponding transfer functions. In other words the transfer functions satisfy equations (10.2). Consider the factored forms:

$$m_0^a(\omega) = \Big(\frac{1+e^{-i\omega}}{2}\Big)^{N_a} p_a(\omega)$$
$$m_0^s(\omega) = \Big(\frac{1+e^{-i\omega}}{2}\Big)^{N_s} p_s(\omega)$$

   Let us define new filters by

$$T^a(\omega) = \Big(\frac{1+e^{-i\omega}}{2}\Big)^{N_a+1} p_a(\omega)$$
$$T^s(\omega) = \Big(\frac{1+e^{-i\omega}}{2}\Big)^{N_s-1} p_s(\omega)$$

   Check that these are also perfect reconstruction filters. This operation is called balancing.

4. Let $m_0^a(\omega) = 1$ and

$$m_0^s(\omega) = \frac{1}{32}\big(-e^{-i3\omega} + 9e^{-i\omega} + 16 + 9e^{i\omega} - e^{i3\omega}\big)$$

   These are called Deslauriers-Dubuc filters. What is N_s in this case? Plot the corresponding scaling functions and wavelets. Then compute the new filters after one and two balancing. Finally plot the corresponding scaling functions and wavelets.