Rotations and powers of complex numbers

Lehman - Pesonen 2007
http://www.joensuu.fi/mathematics/MathDistEdu/SemProd/
ComplexRotationsAndPowers.htm

1. How the successive products $a(e^{iC})^n = ae^{inC}$ rotate a

Other interpretation: iterate $a \mapsto ae^{iC}$.
Play first freely with the dynamic figure (called *sketch*). Push the buttons, move a
in the plane and C on the real argument (radian angle) line.

Problems 1

Reset by pushing the computer keyboard key 'R' and clear traces by clicking the
red cross in the lower right corner (appears only when tracing has been used).
1a) Give values of C between 0 and 2π such that the point ae^{i2C} is on top of a.
Answer 1a1) $C_1 = $ _____
Answer 1a2) $C_2 = $ _____
1b) Give values of C between 0 and 2π such that the point ae^{i3C} is on top of a.
Answer 1b1) $C_1 = $ _____
Answer 1b2) $C_2 = $ _____
Answer 1b3) $C_3 = $ _____
1c) Give values of C between 0 and 2π such that the point ae^{i4C} is on top of a.
Answer 1c1) $C_1 = $ _____
Answer 1c2) $C_2 = $ _____
Answer 1c3) $C_3 = $ _____
Answer 1c4) $C_4 = $ _____

2. Test your geometric ability …

can you imagine a square?

Problems 2

Reset by pushing the key 'R' and clear traces (the red cross, if seen).
You can **mark** a position in the plane by dragging the point a to that position and
clicking the `Trace` and then the `Stop trace` buttons. Then you can move a to
some other position.
First step. Choose a position and mark it (see above).
Second step. There is a square centered at 0 having the marked position as a
vertex. Guess where the other three vertices are; mark these positions.

Third step. Check the accuracy of your intuition using the `Go to` -buttons and a suitable value of C (or C/π).

3. Test your geometric ability …

can you imagine an equilateral triangle?

Problems 3

Reset by pushing the key 'R' and clear traces (the red cross, if seen).
First step. Choose a position and mark it (see above).
Second step. There is an equilateral triangle centered at 0 having the marked position as a vertex. Guess where the other vertices are; mark these positions.
Third step. Check the accuracy of your intuition using the Go to -buttons and a suitable value of C (or C/π).

4. Complete circle or not ?

Parametric curve: $t \mapsto ae^{itC}$

Problems 4

4a) Reset and clear. Put $C/\pi = 1.8$. Click `Go to a*e^i4C`. Click `Fix`, click `Trace`.
Will you trace a complete circle if you click on `Go to a*e^i3C` ?
Answer 4a) _____
Try it.
4b) Clear the traces but do not reset. Will you trace a complete circle if you click on `Go to a*e^iC` ?
Answer 4b) _____
Try it.