EXPERIMENTS ON USING INTERACTIVE WEB-BASED MATHEMATICS PROBLEM SETS BASED ON DYNAMIC GEOMETRY APPLETS

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Background

In this paper we describe our efforts in improving and enriching the teaching and learning of mathematics, especially by using computers. Since 1997, we have had several consecutive projects aiming to increase the role of computer-assisted-learning (CAL), and since 2001 we have conducted a four-year pilot project on applied mathematics, under the umbrella of a larger project called Virtual University of Eastern Finland[1]. Today, most courses include some elements of CAL, in some courses its role is essential and in the future a few cooperatively arranged courses will be practically "virtual", e.g. with video lectures and computerized exercises. These developmental efforts, leading to new ways of teaching and new kinds of learning processes, have also invoked the need of follow-up and research activities. Here we shall introduce a way to use interactive dynamical pictures combined with appropriate problem sets in learning basic mathematical concepts like function, binary operation and eigenvalue.

Difficulties in learning mathematical concepts

According to numerous educational research results the transition from concrete examples to the abstract concept, the process of *abstraction*, is the most demanding step in learning mathematics. The problem has been under serious concern, recently even in research on college and university level mathematics education. Serious difficulties were found in understanding and using definitions of mathematical concepts; most students build their own mental model for the concept, so called *concept image*, by collecting more or less vague attributes and properties of the concept, and tend to use these instead of the *concept definition* itself. This happens often even though they may know and are able to express the exact definition (Vinner 1991). This unfortunate feature may not be so crucial in all mathematics education, but it is, indeed, among mathematics majors who are supposed to achieve a high level in abstraction skills. Of course, it is often convenient and even desirable to be able to use *properties* instead of the perhaps clumsy definition, but for understanding and building a theory this is certainly insufficient.

For the concept of function, Tall & Bakar (1991) conclude that "the learner cannot construct the abstract concept of function without experiencing examples of the function concept in action, and the students cannot study examples of the function concept in action without developing prototype examples having built-in limitations that do not apply to the abstract concept". Support for this statement can be found in many other studies as Breidenbach *et al*. (1992), Tall (1992), Vinner & Dreyfus (1989), and Brown *et al*. (1997).

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[1] The participants are the Universities in Joensuu, Kuopio and Lappeenranta.

A cautious teacher lets the students to be faced with problems which *require* the use of definitions, as well as with problems that can be solved using only properties. For example in the case of the function concept we can vary the domain, co-domain and the rule itself, in order to produce a great variety of examples of functions and non-functions.

A related problem is the dilemma between procedural and conceptual knowledge: Does the student have to *understand* for being able to *do*, and vice versa? A comprehensive theoretical framework for the approaching to this basic question can be found in Haapasalo & Kadijevich (2000), or Kadijevich & Haapasalo (2001), as far as CAL is concerned.

On the role of computer-based activities

With computers it is possible to construct activities that combine the *doing* and the *understanding*, possibly equipped with automatized control of success. Another advantage is that working around a computer drives students in lively discussions, and having to do and think makes them much better engaged in the work than ordinary exercise sessions (but there are also counterexamples: some students don't like struggling with computers, and some get easily frustrated if they are not able to solve the problems immediately!).

One remarkable attempt in this activating direction is a computer-based educational framework based on *reflective abstraction*, introduced and used successfully by Ed Dubinsky and his colleagues see e.g. Asiala *et al.* (1996). The abstraction process is meant to develop via action-process-object-schema path, using rather concrete computer manipulations of mathematical object, mainly numerical or symbolical expressions, and here an important dimension is the use of cooperative learning in small 2-4 person groups. We have used this method in abstract algebra and discrete mathematics courses, partially also in linear algebra.

In this paper we deal with another particular approach: working with multiple representations of a mathematical concept, which is supposed to help significantly in the abstraction process. For example, in learning the concept of function or binary operation we can use the interplay between verbal, symbolical and graphical representations (VSG, see Haapasalo 1993, 1997).

Unfortunately, it is often hard to see the functioning of a binary operation or, more generally, a two variable function, from sole static pictures. Therefore we have started to use dynamical, mouse dragable pictures, which allow the student to interact directly with the mathematical construction and which show nicely the process nature of the concept.

Pedagogical Background

In most activity cases we try to implement a (version of) five-step model of concept formation process, based on constructivist view of knowledge and learning. According to this model, the learning of a concept should be arranged in the following steps:

1) Orientation: organize suitable learning situations, provoke need to learn.
2) Definition: starting from collecting a vague set of properties, strangling towards the very definition.
3) Identification: building capability to recognize similarities/differencies between a model and the definition.
4) Production: maturing in building own examples, realizations and alternative ways to represent the concept.

5) Reinforcement: making the understanding sure, for example via applications.

Especially the identification step is usually poorly appreciated, although it has appeared crucial in several repeated investigations. On the other hand, the production phase is known to indicate most reliably the depth of knowledge and skill.

Here we concentrate to describe multiple VSG representations using statical and dynamical identification and production problems, while in real courses we use also many other kinds of tasks. For example, in learning abstract binary operations we use relations or rules defined on pairs of alphabets, e.g. restricted to vowels.

In the identification phase (I) one should use all possible combinations IVS (identifying between verbal and symbolical forms), IVG, ISG, and in production phase (P) at least all those variations PXY, where (different) X and Y can be V, S or G. A more detailed discussion about the framework and former experiment results in school level is found in Haapasalo (1993, 1997).

Many of these tasks are easily implemented in computer based material, and even checked or tutored automatically. In distance learning, these can help the student to check the acquired knowledge oneself. Also, these tasks are ideal to be studied cooperatively around the computer. However, it is not evident that an average freshman can manage alone without a local tutor or at least a supporting engaged peer group. We noticed soon that more comprehensive studies concerning how the students work with these problem sets and what kind of guidance and tutoring would be necessary.

This kind of electronic form learning materials should be suitable for upper secondary, college and university level mathematics. Ways to produce and to use such materials are meant to be suitable for pre-service (and also in-service) mathematics teacher education, which is an important task of the University of Joensuu. Students are encouraged to learn how to make such material even themselves, but not very widely yet.

About the Techniques

Today the World Wide Web documents can be reached from anywhere, they can contain animations and simulations, and many kinds of interaction between the user and the document is possible. This enables the same material to be used as well in computerized classroom as in distance education. For example, Maple generated animations are sequences of pictures, like a movie, and they are most useful in process illustrations, but not so convenient for more demanding interactions.

A Java applet can reside in a server and it can be invoked by a remote document or user. With dynamical geometry Java applets like JavaSketchpad and Geometria one can construct interactively controllable pictures (sketches), in which mathematical elements like points, segments, lines, circles and objects obtained by geometrical transformations from them. Controlling may be e.g. dragging points by the computer mouse, or pushing control buttons to show or hide objects, or to invoke movements or animations. In Geometria applet construction one can also ask for evaluating the present state of the picture, and receive the comments or advice that the creator has programmed in it. This feature in Geometria is called *response analysis*. The produced sketch codes can be embedded in any html code between the tags `<applet>` and `</applet>`, and Java supporting browsers can draw the sketch on the

display. Dynamical sketches can also be easily used inside WebCT-managed course material and tests. More about the techniques can be found at URL
http://www.joensuu.fi/mathematics/MathDistEdu/MAA2001/index.html

The Javasketcpad applet is property of Key Curriculum Press, and it is downloadable and documented at Geometer's Sketchpad www pages. If fact, one can also use Geometer's Sketchpad to generate the applet sketches, but we have used mainly manual coding. The Geometria applet, built originally by David Joyce and developed further by Timo Ehmke, is described thoroughly in the dissertation of Ehmke (2001), together with some related experimental research results. Both applets are free for educational use.

Examples of interactive worksheets

Several interactive www documents have been created and tried in classroom teaching, as well as distance learning material. The documents contain (hyper)text, multiple choice and puzzle-like problems, possibly enriched with JavaSketchpad or Geometria based sketches. Some problems can be auto-checked by the system, open ended answers are sent as email by the www form to the teacher.

Some interactive material for courses Analysis 1, Complex Analysis, Discrete Mathematics Linear Algebra, Pre-Calculus and Plane Geometry and have been tested in classroom or as homework. In the Figure we see a snapshot from the document Function Demo. In the Figure the tracing facility is being used when ***x*** moves to the right.

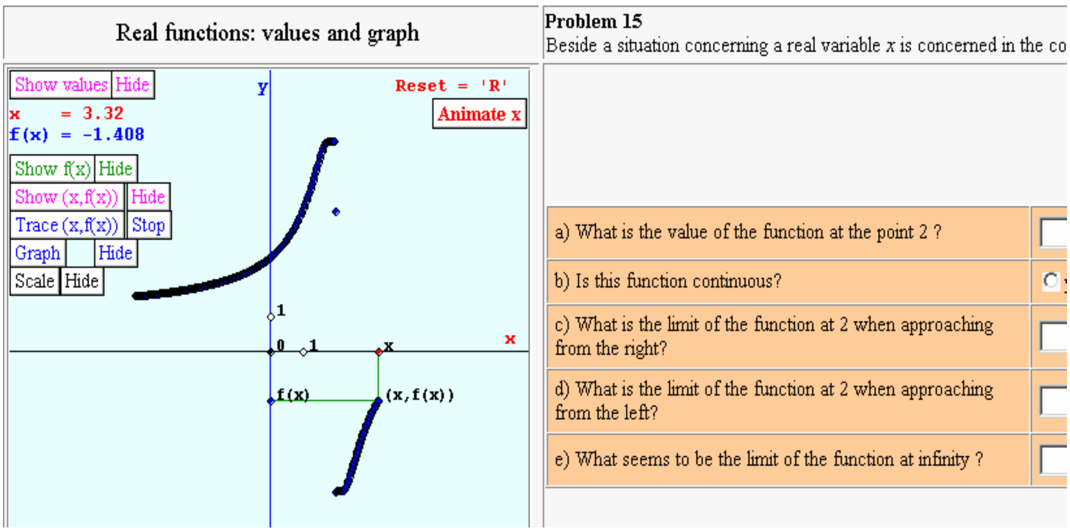

***Figure. 1.* Java Sketchpad sketch on a www form.**

Most of the materials are reached through URL
http://www.joensuu.fi/mathematics/MathDistEdu/SemProd/AnimationCatalogue.html

Research components

We made a short experiment on using these dynamical Java Sketchpad applet constructions in the Linear Algebra Spring 2000 course, and again slightly wider in the 2001 course. The www worksheets deal with binary operation, scaling functions, linear space and basis and

linear transforms in the Euclidean plane (parts I and II). The linear transform document Part I equipped with notes on students' success and comments can be found at URL
http://www.joensuu.fi/mathematics/MathDistEdu/Helsinki2002/index.html
The Linear Algebra course text (in Finnish), now being implemented in hypertext form, also contains numerous dynamical illustrations, see URL
http://www.joensuu.fi/matematiikka/kurssit/LinAlg/Kurssimateriaali/

Hanna Lehtola made her Master's Thesis (2002) and some related studies on the use of Java Sketchpad applet constructions in Analysis and Linear Algebra. Analysis freshmen tested in Autumn 2001 the www worksheet Function Demo (Figure), which contains various graphical representations about the function concept. Lehtola analyzed the student products, and in Pesonen *et al.* (2003) this experiment was summarized and interpreted in the light of the interplay between procedural and conceptual knowledge. Hyperlinks to the tests and an English version of the Function Demo equipped with student scores are found at URL
http://www.joensuu.fi/mathematics/MathDistEdu/Paper2003/FunctionConcept.htm

Based on her studies Lehtola constructed a revised version of identification activities concerning the function concept. Furthermore, she constructed interactive material for Discrete Mathematics and these were tested in autumn 2002, partly by mathematics freshmen, partly by Discrete Mathematics course participants, see URL
http://www.joensuu.fi/mathematics/MathDistEdu/Kuopio2003/Kuopio12303.htm
Many of the Java Sketchpad worksheets were preliminarily tested in the University of Caen in November 2002, and more thorough testing is planned to take place in November 2003.

A new ongoing study concerning the use of dynamical sketches is supported 2003-2004 by the Academy of Finland and DAAD in Germany. It was started in autumn 2002 by a pre-study experiment in Joensuu and the research material is being analysed by Ehmke in the Leibniz Institute in Kiel. The research problem is about how the students really act when struggling with the interactive problems. The research material consists of screen-captured video files recorded from students' work. The goal is to produce a classification system for the different ways students play with the worksheets. This is supposed to help in the design of more functional material. Some partial results of this study will be reported in the conference talk.

Literature, used in the article

Asiala, M., Brown A., DeVries D., Dubinsky, Ed, Mathews, D. & Thomas, K. 1996. A Framework for Research and Curriculum Development in Undergraduate Mathematics Education. *Research in Collegiate Mathematics Education II*, 3, 1-32.

Breidenbach, D., Dubinsky, E., Hawks, J. & Nichols, D. 1992. Development of the process conception of function. *Educational Studies in Mathematics* 23, 247-285.

Brown, A., De Vries, D., Dubinsky, E. & Thomas, K. 1997. Learning binary operations, groups, and subgroups. *Journal of Mathematical Behavior* 16 (3), 187-239.

Ehmke, T. 2001: Eine Klasse beweglicher Figuren für interaktive Lernbausteine zur Geometrie. *Dissertation*. Universität Flensburg, URL:http://www.uni-flensburg.de/mathe/homepg/schreiber/download/diss_ehmke.pdf

Haapasalo, L. 1993. Systematic constructivism in mathematical concept building. In P. Kupari & L. Haapasalo (eds.), *Constructivist and Curriculular Issues in the Finnish School Mathematics Education.* Mathematics Education Research in Finland. Yearbook 1992-1993. University of Jyväskylä, Institute for Educational Research.

Publication Series B 82.
Haapasalo, L. 1997. Planning and assessment of construction processes in collaborative learning. In S. Järvelä & E. Kunelius (eds.) *Learning & Technology - Dimensions to Learning Processes in Different Learning Environments*. Electronic publications of the pedagogical faculty of the University of Oulu. Internet: http://herkules.oulu.fi/isbn9514248104
Haapasalo, L. & Kadijevich, Dj. 2000. Two types of mathematical knowledge and their relation. *Journal für Mathematikdidaktik* 21, 139-157.
Kadijevich, Dj. & Haapasalo, L. 2001. Linking procedural and conceptual mathematical knowledge through CAL. *Journal of Computer Assisted Learning* 17, 156-165.
Lehtola, H. 2002. Vuorovaikutteiset matemaattiset animaatiot käsitteiden oppimisessa. (Interactive animations in learning of concepts). *Master's Thesis.* University of Joensuu, Department of Mathematics.
Pesonen, M. E., Haapasalo, L. & Lehtola, H. 2003. Looking at Function Concept through Interactive Animations. *The Teaching of Mathematics* 5, 37-45.
Tall, D. 1992. The transition to advanced mathematical thinking: functions, limits, infinity and proof. In D. Grouws (ed.) *Handbook of research on mathematics teaching and learning*. NY: MacMillan, 495-511.
Tall, D. & Bakar, M. 1991. Students' Mental Prototypes for Functions and Graphs. Downloadable on Internet at http://www.warwick.ac.uk/staff/David.Tall/pdfs/dot1991f-mdnor-function-pme.pdf
Vinner, S. 1991. The role of definitions in teaching and learning. In D. Tall (ed.): *Advanced mathematical thinking* (pp.). Dordrecht: Kluwer, 65-81
Vinner, S. & Dreyfus, T. 1989. Images and definitions for the concept of function. *Journal for Research in Mathematics Education* 20, 356-366.